Regular paper

NUMERICAL METHODS FOR DESIGNING IIR FILTERS WITH EQUIRIPPLE APPROXIMATION OF THE CONSTANT GROUP DELAY

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Abstract: A design method for recursive digital filters is presented which yields to the filters with equripple approximation of the constant group delay. This approximation is applied to the transfer function having n-th order zero in the first case at the origin and in second case at the $z=-1$, where n is the filter order. The equiripple solution is obtained with Remez exchange algorithm. For solving the set of nonlinear equations numerical methods are discussed.

Keywords: Equiripple approximation, numerical methods, Remez exchange algorithm.

INTRODUCTION

In application such as pulse transmission and high-speed data transmission in addition to specified magnitude characteristic, it is desired that filters have also linear phase characteristics. The problem of designing a stable recursive digital filter (IIR) to have linear phase or constant group delay characteristics may be considered as an direct approximation in digital domain ([1-4]). The well known Remez exchange algorithm [5] is used for numerical solution of a set of nonlinear equations. The aim is to adjust the poles position of transfer function so that group delay is equripple. It is shown that there are two possible types of these filters. In the first case the transfer function has n-th order zero at the origin (type I)and in the second case at the $z=-1$ (type II), where n is the filter order.

APPROXIMATION TECHNIQUE

The transfer function of the recursive digital filter can be written in the following form

$$F_n(z)=\frac{h_0(z+\nu)^n}{(z-r_0)^{\mu}\prod_{i=1}^{m}\left(z-r_i\,\mathrm{e}^{\mathrm{j}\varphi_i}\right)}=\frac{h_0\left(1+\nu z^{-1}\right)^n}{\sum_{k=0}^{n}a(k)z^{-k}} \qquad (1)$$

where $n=\mu+m$ is the filter order (m is an even number since the poles appear in the conjugated complex pairs), h_0 is a real constant selected so as to satisfy the conditions $|F_n(1)|=1$. Depending on the value ν (0 or 1), there are two possible types of recursive digital filters.

For n even, $\mu=0$ while for n odd, $\mu=1$ is chosen. For $\nu=0$ function has n-th order zero located at the origin and for $\nu=1$ located at the $z=-1$. The corresponding group delay response is obtained by substituting $\mathrm{e}^{\mathrm{j}\theta}$ for z into equation (2), where $\theta=\omega T$, thus giving

$$\tau_n(\theta)=-\operatorname{Re}\left\{z\frac{\mathrm{d}}{\mathrm{d}z}\left[\ln F(z)\right]\right\}\Big|_{z=\mathrm{e}^{\mathrm{j}\theta}} \qquad (2)$$

which corresponds to

$$\tau_n(\theta)=-\frac{n}{\nu+1}+\mu\frac{1-r_0\cos\theta}{1-2r_0\cos\theta+r_0^2}+\sum_{i=1}^{m}\frac{1-r_i\cos(\theta-\varphi_i)}{1-2r_i\cos(\theta-\varphi_i)+r_i^2} \qquad (3)$$

where $r_{2i}=r_{2i-1}$ and $\varphi_{2i}=-\varphi_{2i-1}$ for $i>0$.

Using equiripple approximation it is important to consider the number of local maxima and minima of group delay in the range $0\le\theta\le\pi$. Equation (1) is n-th order transfer function, and this means that there can only be n local maxima and minima, θ_k, $k=1,2,\ldots,n$, in the range $0\le\theta\le\pi$.

An equiripple approximation constant group delay to a lowpass 5-th order filter designed using mini-max approximation of group delay is shown in Fig.1, and it is seen that this approximation requires approximating to constant group delay τ_d with maximum error $\varepsilon_{\max}$. Note that $\tau_n(\theta)$ has local maximum at $\theta=0$.

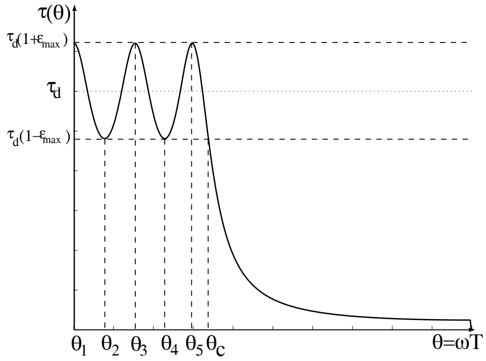

Fig. 1 – *Group delay of the 5-th order recursive digital filter for* $\varepsilon_{\max}=20\%$.

In designing recursive digital filters using approximation of constant group delay in an equripple manner, it is not possible to specify independently each of the filter

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parameters $\varepsilon_{\max}$, $\tau_{\boldsymbol{d}}$, cutoff frequency θ_{c} and order of the filter n . One approach is to fix the values $\varepsilon_{\max}$, $\tau_{\boldsymbol{d}}$ and n and let θ_{c} be variable.

If constant group delay $\tau_{\boldsymbol{d}}$ is approximated in the Chebyshev sense with maximum error $\varepsilon_{\max}$ then the unknown vector $\boldsymbol{x}$

$$\boldsymbol{x} = \left[r_0, r_1, \varphi_1, \ldots, r_{m/2}, \varphi_{m/2}, \theta_2, \ldots, \theta_n\right]^{\mathrm{T}} \tag{4}$$

which contains n pole locations, $r_0, r_1, \varphi_1, \ldots, r_{m/2}, \varphi_{m/2}$, and n - 1 frequencies at which extremes occur, $\theta_2, \ldots, \theta_n$, can be determined by solving the following systems of $2n$ - 1 nonlinear equations:

$$\tau_n(\theta_{\boldsymbol{k}}) = \left[1 + (-1)^{k+n}\varepsilon_{\max}\right]\tau_{\mathrm{d}} \;\text{ for }\; k = 1, 2, \ldots, n; \tag{5}$$

and

$$\frac{\partial \tau_n(\theta)}{\partial \theta}\Big|_{\theta=\theta_k} = \mu \frac{r_0\left(r_0^2 - 1\right)\sin\theta_k}{\left(1 - 2r_0\cos\theta_k + r_0^2\right)^2} + \sum_{i=1}^{m} \frac{r_i\left(r_i^2 - 1\right)\sin(\theta_k - \varphi_i)}{\left(1 - 2r_i\cos(\theta_k - \varphi_i) + r_i^2\right)^2} = 0 \tag{6}$$

for $k = 2, 3, \ldots, n$, since $\frac{\partial \tau_n(\theta)}{\partial \theta}\Big|_{\theta=\theta_1} \equiv 0$ because $\tau_n(\theta)$ is even function.

The equation (6) represents the first derivation of the equation (3) with respect to θ , where θ_k , $k = 1, 2, \ldots, n$ are extremes or critical points at which the maximum relative delay error $\varepsilon_{\max}$ occurs.

Thus it can be seen from equation (5) that $\tau(\theta)$ will have either a local maximum for n even and local minimum for n odd at $\theta = 0$.

The set of $2n$ - 1 non-linear equations (5) and (6) can be expressed as

$$f_j(\boldsymbol{x}) = 0 \,,\; j = 1, \ldots, 2n\text{-}1 \tag{7}$$

where

$$f_j(\boldsymbol{x}) = \begin{cases} \tau_n(\theta_k) - \left[1 + (-1)^{k+n}\varepsilon_{\max}\right]\tau_{\mathrm{d}}, & k = 1, 2, \ldots, n \\ \dfrac{\partial \tau_n(\theta)}{\partial \theta}\Big|_{\theta=\theta_k}, & k = 2, 3, \ldots, n, \end{cases} \tag{8}$$

first and second expression, for $k = 1, 2, \ldots, n$, gives elements of $f_j(\boldsymbol{x})$ for $j = 1, 2, \ldots, n$, and second, for $k = 2, 3, \ldots, n$, gives elements of $f_j(\boldsymbol{x})$ for $j = n+1, \ldots, 2n\text{-}1$.

Unfortunately the $2n$ - 1 equations are nonlinear and must be solved by an iterative process.

The Remez exchange algorithm [4] for solving these equations is used. Taking $\boldsymbol{x}(l)$ to be current value of $\boldsymbol{x}$ and $\boldsymbol{x}(l+1)$ to be the new value of $\boldsymbol{x}$ obtained after one iteration, the relations between the current values of $\boldsymbol{x}$, $\boldsymbol{x}(l)$, and the new values of $\boldsymbol{x}$, $\boldsymbol{x}(l+1)$, may be expressed in the matrix form

$$\begin{bmatrix} x_1(l+1) \\ x_2(l+1) \\ \vdots \\ x_{2n-1}(l+1) \end{bmatrix} = \begin{bmatrix} x_1(l) \\ x_2(l) \\ \vdots \\ x_{2n-1}(l) \end{bmatrix} - \lambda \begin{bmatrix} \frac{\partial f_1(l)}{\partial x_1} & \frac{\partial f_1(l)}{\partial x_2} & \cdots & \frac{\partial f_1(l)}{\partial x_{2n-1}} \\ \frac{\partial f_2(l)}{\partial x_1} & \frac{\partial f_2(l)}{\partial x_2} & \cdots & \frac{\partial f_2(l)}{\partial x_{2n-1}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_{2n-1}(l)}{\partial x_1} & \frac{\partial f_{2n-1}(l)}{\partial x_2} & \cdots & \frac{\partial f_{2n-1}(l)}{\partial x_{2n-1}} \end{bmatrix}^{-1} \begin{bmatrix} f_1(l) \\ f_2(l) \\ \vdots \\ f_{2n-1}(l) \end{bmatrix} \tag{9}$$

where λ improves the solution convergence and satisfies the condition $0 < \lambda \le 1$. In the beginning of the iterative procedure $\lambda = 0.1$ should be chosen. When condition $\max\{f_j(l)\} \le 10^{-4}$ is fulfilled, then $\lambda = 1$ should be taken.

In the equation (9) elements $\partial f_j / \partial x_s$, $s = 1, \ldots, 2n\text{-}1$, when $f_j = \tau_n(\theta_k)$, $k = 1, \ldots, n$, have form

$$\frac{\partial f_j}{\partial r_0} = \frac{\left(1 + r_0^2\right)\cos\theta_k - 2r_0}{\left(1 - 2r_0\cos\theta_k + r_0^2\right)^2}\,; \tag{10}$$

for $i = 1, \ldots, m/2$ derivations are:

$$\frac{\partial f_j}{\partial r_i} = \frac{\left(1 + r_i^2\right)\cos(\theta_k - \varphi_i) - 2r_i}{\left(1 - 2r_i\cos(\theta_k - \varphi_i) + r_i^2\right)^2} + \frac{\left(1 + r_i^2\right)\cos(\theta_k + \varphi_i) - 2r_i}{\left(1 - 2r_i\cos(\theta_k + \varphi_i) + r_i^2\right)^2} \tag{11}$$

$$\frac{\partial f_j}{\partial \varphi_i} = \frac{-r_i\left(r_i^2 - 1\right)\sin(\theta_k - \varphi_i)}{\left(1 - 2r_i\cos(\theta_k - \varphi_i) + r_i^2\right)^2} + \frac{-r_i\left(r_i^2 - 1\right)\sin(\theta_k + \varphi_i)}{\left(1 - 2r_i\cos(\theta_k + \varphi_i) + r_i^2\right)^2} \tag{12}$$

$$\frac{\partial f_j}{\partial \theta_k} = \mu \frac{r_0\left(r_0^2 - 1\right)\sin\theta_k}{\left(1 - 2r_0\cos\theta_k + r_0^2\right)^2} + \sum_{i=1}^{m/2} \frac{r_i\left(r_i^2 - 1\right)\sin(\theta_k - \varphi_i)}{\left(1 - 2r_i\cos(\theta_k - \varphi_i) + r_i^2\right)^2} + \sum_{i=1}^{m/2} \frac{r_i\left(r_i^2 - 1\right)\sin(\theta_k + \varphi_i)}{\left(1 - 2r_i\cos(\theta_k + \varphi_i) + r_i^2\right)^2} \tag{13}$$

$$\frac{\partial f_j}{\partial \theta_s} = 0 \;\text{ for }\; s \ne k\,. \tag{14}$$

Expressions (10)-(14) form $1, \ldots, n$ rows of matrix above.

Elements $\partial f_j / \partial x_s$, $s = 1, \ldots, 2n\text{-}1$, when $f_j = \partial \tau_n(\theta)/\partial \theta\big|_{\theta=\theta_k}$, $k = 2, \ldots, n$, have form

$$\frac{\partial f_j}{\partial r_0} = -\frac{\left(1 - 6r_0^2 + r_0^4\right)\sin\theta_k + r_0\left(1 + r_0^2\right)\sin 2\theta_k}{\left(1 - 2r_0\cos\theta_k + r_0^2\right)^3} \tag{15}$$

for $i = 1, \ldots, m/2$ derivations are:

$$\frac{\partial f_j}{\partial r_i} = -\frac{\left(1 - 6r_i^2 + r_i^4\right)\sin(\theta_k - \varphi_i) + r_i\left(1 + r_i^2\right)\sin 2(\theta_k - \varphi_i)}{\left(1 - 2r_i\cos(\theta - \varphi_i) + r_i^2\right)^3} - \frac{\left(1 - 6r_i^2 + r_i^4\right)\sin(\theta_k + \varphi_i) + r_i\left(1 + r_i^2\right)\sin 2(\theta_k + \varphi_i)}{\left(1 - 2r_i\cos(\theta_k + \varphi_i) + r_i^2\right)^3} \tag{16}$$

$$\frac{\partial f_j}{\partial \varphi_i} = -\frac{r_i\left(r_i^2-1\right)\left[\left(r_i^2+1\right)\cos\left(\theta_k-\varphi_i\right)+r_i\left(\cos 2\left(\theta_k-\varphi_i\right)-3\right)\right]}{\left(1-2r_i\cos\left(\theta_k-\varphi_i\right)+r_i^2\right)^3} - \frac{r_i\left(r_i^2-1\right)\left[\left(r_i^2+1\right)\cos\left(\theta_k+\varphi_i\right)+r_i\left(\cos 2\left(\theta_k+\varphi_i\right)-3\right)\right]}{\left(1-2r_i\cos\left(\theta_k+\varphi_i\right)+r_i^2\right)^3} \qquad (17)$$

and

$$\frac{\partial f_j}{\partial \theta_k} = \mu\frac{r_0\left(r_0^2-1\right)\left[\left(r_0^2+1\right)\cos\theta_k+r_0\left(\cos 2\theta_k-3\right)\right]}{\left(1-2r_0\cos\theta_k+r_0^2\right)^3} + \sum_{i=1}^{m/2}\frac{r_i\left(r_i^2-1\right)\left(\left(r_i^2+1\right)\cos\left(\theta_k-\varphi_i\right)+r_i\left(\cos 2\left(\theta_k-\varphi_i\right)-3\right)\right)}{\left(1-2r_i\cos\left(\theta_k-\varphi_i\right)+r_i^2\right)^3} + \sum_{i=1}^{m/2}\frac{r_i\left(r_i^2-1\right)\left(\left(r_i^2+1\right)\cos\left(\theta_k+\varphi_i\right)+r_i\left(\cos 2\left(\theta_k+\varphi_i\right)-3\right)\right)}{\left(1-2r_i\cos\left(\theta_k+\varphi_i\right)+r_i^2\right)^3} \qquad (18)$$

$$\frac{\partial f_j}{\partial \theta_s} = 0 \text{ for } s \neq k . \qquad (19)$$

Expressions (15)-(19) form $n+1,\ldots,2n-1$ rows of matrix above. Note that $x_1 \equiv r_0$, $x_2 \equiv r_1$, $x_3 \equiv \varphi_1$, $x_4 \equiv r_2$, etc.

Initial conditions

To make a starting guess at the solution is the most important part in solving this equations system. The initial conditions are adopted as follows. For moduli $r_0, r_1, \ldots, r_{m/2}$ the distances close to the unit circle are taken, for example $r_0 = r_1 = \ldots = r_{m/2} = 0.9$, for angles $\varphi_1, \ldots, \varphi_{m/2}$ are taken $\varphi_i = \pi/4 + i\,\pi/8$, $i = 0,\ldots,m/2$, for τ_d is taken the same value as the order of the filter n. Because group delay local maximum occur at the pole angle frequencies φ_i, for frequencies $\theta_2, \ldots, \theta_n$ are taken the same values as the poles angles for maximums of the group delay and the values between two maximums for minimums.

Numerical methods

For this numerical method it is necessary to use all expressions (10)-(19), to form matrix for iterative procedure described by equation (9). This algorithm can be applied in any program package but this program can be very complicated for high filter order. Already seventh order filter is too long to program. Since, there are different program packages which contain subroutines for solving the system of nonlinear equations, it is recommendable to use this convenience for higher order filter.

Using command 'fsolve', generated in MATLAB software, only equations (3) and (6) are enough to find the solution of system of nonlinear equations (7), written in matrix form as

$$f_j(\boldsymbol{x}) = \left[f_1(\boldsymbol{x}), f_2(\boldsymbol{x})\ldots, f_{2n\text{-}1}(\boldsymbol{x})\right]$$

which will be marked in program as

$$F = \left[F(1), F(2)\ldots, F(2n-1)\right]$$

In general, this command can be used in two cases.

In the first case, the system of 2n-1 nonlinear equations can be defined as subroutine, in separate M-file with arbitrary name 'myfun', as matrix F:

```
function F = myfun(x)
F =[F(1);F(2);…F(2n -1)].
```

Then, in the main M-file, the initial conditions are defined with matrix x0, and 'fsolve' command gives the solution for vector given by equation (4):

```
x0 = [x0(1);x0(2);…x0(2n-1)];
x = fsolve(@myfun,x0).
```

In the second case, the system of nonlinear equations F(x)=0 can also be defined as inline object in main M-file as

```
F=inline('[F(1);F(2);…F(2n-1)]')
x0 = [x0(1);x0(2);…x0(2n-1)];
x = fsolve(F,x0);
```

There is important difference between this cases. If in the main M-file some parameters are defined, which exist in subroutine (the first case), this subrotine will not accept this values as known and it is not possible to solve this problem except using the "inline" command (the second case).

EXAMPLES

As an application of this method two following examples are given for the eleventh order filter. Fig.1 represents characteristic of transfer function having n-th order zero at the $z=-1$, $(\nu = 1)$, and in Fig.2 having n-th order zero at the origin, $(\nu = 0)$.

First example

The adopted initial conditions are: $\tau_d = 11$,

$r_0 = r_1 = r_2 = r_3 = r_4 = r_5 = 0.85$, for poles angles, $\varphi_k = k(\pi/8), k = 1,\ldots,5$ and for frequencies $\theta_k = k(\pi/16)$, $k = 1,\ldots,10$. Since $\nu = 0$ function has n-th order zero located at the $z = -1$. The maximum error is $\varepsilon_{\max} = 20\%$.

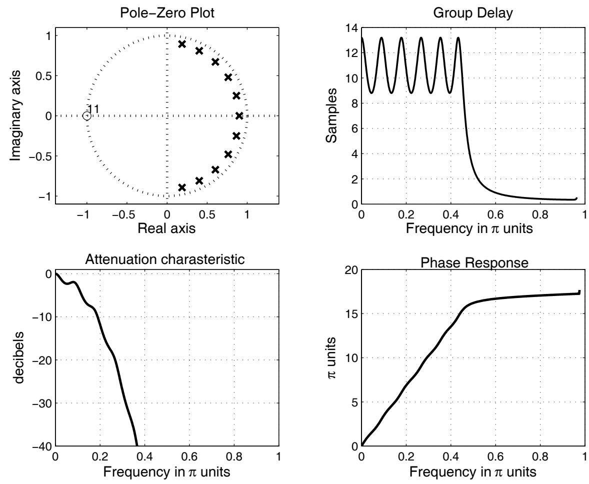

Fig. 2 – *Group delay, Pole zero plot, Phase response and Attenuation characteristic for 11-th order recursive digital filter for* $\nu = 1$, $\tau_d = 11$, $\varepsilon_{\max} = 20\%$.

Second example

The adopted initial conditions are the same as for the first example. Since $\nu = 1$, function has n-th order zero located at the $z = 0$.

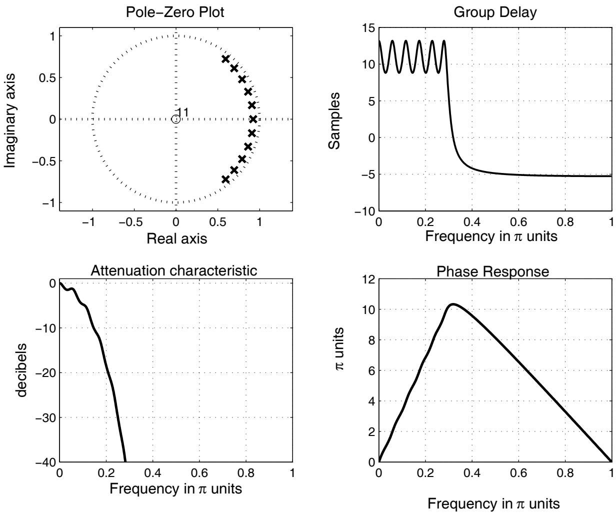

Fig. 3 – *Group delay, Pole zero plot, Phase response and Attenuation characteristic for 11-th order recursive digital filter for* $\nu = 0$, $\tau_d = 11$, $\varepsilon_{\max} = 20\%$.

Note that, in the first example, shown in the Fig.1, angle response function is only increasing function and in the second example, shown in the Fig.2, angle response has increasing and decreasing part so it has maximum.

Calculated coefficients $a(k)$ and h_0 for 11-th filter order, equation (1), are shown in the Table I. In order to practical implementation, the pole locations and frequencies at wich group delay local extremums occur are shown in the Table II for $n = 11$. Pole angles, φ_i, and frequencies, θ_i, are normalized with π. Tables contain the first case $(\nu = 0)$ solutions and the second case $(\nu = 1)$ solutions. The group delay maximum errors are 20% and 5%.

Table I

Calculated $a(k)$ *coefficients and constant* h_0 *for different values of* $\varepsilon_{\max}$ *for* 11*-th order filter.*

	$\nu = 1$	
$\varepsilon_{\max}$	20%	5%
$a(0)$	1.0000	1.0000
$a(1)$	-6.5414	-6.4981
$a(2)$	21.2941	20.8352
$a(3)$	-45.3310	-43.3265
$a(4)$	69.8949	64.7221
$a(5)$	-81.7850	-72.7663
$a(6)$	74.0207	62.7445
$a(7)$	-51.8062	-41.4712
$a(8)$	27.4967	20.5953
$a(9)$	-10.5579	-7.3267
$a(10)$	2.6467	1.6837
$a(11)$	-0.3295	-0.1899
h_0	$1.0009 \cdot 10^{-6}$	$9.6739 \cdot 10^{-7}$

	$\nu = 0$	
$\varepsilon_{\max}$	20%	5%
$a(0)$	1.0000	1.0000
$a(1)$	-8.6217	-8.4678
$a(2)$	34.9741	33.6467
$a(3)$	-88.0504	-82.7518
$a(4)$	152.7815	139.8864
$a(5)$	-191.7739	-170.5824
$a(6)$	177.6485	153.0717
$a(7)$	-121.4370	-101.0606
$a(8)$	60.0360	48.1065
$a(9)$	-20.4478	-15.7257
$a(10)$	4.3198	3.1779
$a(11)$	-0.4291	-0.3009
h_0	$3.1532 \cdot 10^{-5}$	$3.4347 \cdot 10^{-5}$

Table II

Calculated vector $\boldsymbol{x}$ *elements for different values of* $\varepsilon_{\max}$ *for the 11-th order filter.*

	$\nu = 1$		$\nu = 0$	
$\varepsilon_{\max}$	20%	5%	20%	5%
r_0	0.9011	0.8517	0.9230	0.8896
r_1	0.9012	0.8520	0.9231	0.8898
φ_1	0.2810	0.2688	0.1828	0.1762
r_2	0.9015	0.8530	0.9234	0.8907
φ_2	0.5611	0.5360	0.3648	0.3512
r_3	0.9022	0.8552	0.9242	0.8927
φ_3	0.8387	0.7993	0.5450	0.5233
r_4	0.9040	0.8609	0.9261	0.8977
φ_4	1.1106	1.0541	0.7207	0.6892
r_5	0.9127	0.8826	0.9345	0.9157
φ_5	1.3649	1.2873	0.8836	0.8407
θ_2	0.1405	0.1344	0.0914	0.0881
θ_3	0.2810	0.2686	0.1828	0.1760
θ_4	0.4212	0.4024	0.2739	0.2636
θ_5	0.5610	0.5355	0.3647	0.3508
θ_6	0.7002	0.6677	0.4551	0.4371
θ_7	0.8385	0.7984	0.5448	0.5224
θ_8	0.9754	0.9267	0.6333	0.6059
θ_9	1.1101	1.0512	0.7200	0.6865
θ_{10}	1.2402	1.1684	0.8032	0.7617
θ_{11}	1.3605	1.2696	0.8786	0.8253

This values can be used for making initial solution for the new filter design.

It can be noticed that as the $\varepsilon_{\max}$ decreases, the group delay linearization bandwidth decreases too.

CASCADE REALIZATION

The results, given above, are obtained using double precision floating-point arithmetic, but this precision can't be achieved in practical realization. Therefore, the calculated coefficients for the cascade realization, in binary representation, are rounded to digital word having length of 2 bits for integer part, and 8 bits for the fractional part of the number. It is found that this rounding is satisfactorily, and group delay, pole zero plot, phase response and attenuation characteristics almost matching with those before rounding .

Cascade realization of transfer function (1) has form

$$F_{11}(z) = h_0 \prod_{k=1}^{6} H_k(z) = h_0 \prod_{k=1}^{6} \frac{b_{0k} + b_{1k} z^{-1} + b_{2k} z^{-2}}{1 + a_{1k} z^{-1} + a_{2k} z^{-2}} .$$

Calculated coefficients for the second-order section form of the 11-th order filter, for the maximum error $\varepsilon_{\max} = 20\%$, for the first ($\nu = 0$) and the second type ($\nu = 1$) are shown in the Table III.

Table III

Calculated coefficients for the second-order section form of the 11th order filter transfer function, for $\varepsilon_{\max} = 20\%$ *and* $\nu = 0$.

b_0	b_1	b_2	a_1	a_2
1.0000	0	0	-0.9258	0
1.0000	0	0	-1.8164	0.8516
1.0000	0	0	-1.7266	0.8516
1.0000	0	0	-1.5820	0.8516
1.0000	0	0	-1.3945	0.8555
1.0000	0	0	-1.1875	0.8711

Calculated coefficients for the second-order section form of the 11th order filter transfer function, for $\varepsilon_{\max} = 20\%$ *and* $\nu = 1$.

b_0	b_1	b_2	a_1	a_2
1.0000	0.9414	0	-0.9023	0
1.0000	1.9961	1.1328	-1.7344	0.8086
1.0000	1.9961	1.0859	-1.5273	0.8125
1.0000	1.9961	1.0117	-1.2070	0.8125
1.0000	1.9375	0.9414	-1.8047	0.8164
1.0000	1.8945	0.8984	-1.3750	0.8320

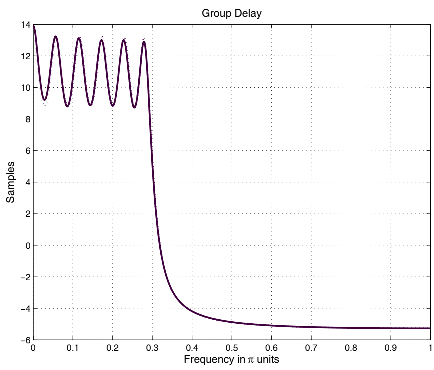

Fig.4 – *Group delay for the 11-th order recursive digital filter for* $\nu = 0$ *and* $\varepsilon_{\max} = 20\%$ *before coefficients rounding (dotty line) and after rounding (solid line).*

In Fig.4 it is shown that the group delay deviation, after the second-order section form coefficients rounding, in regard to ideal group delay characteristic, is negligible for the chosen digital word length.

CONCLUSION

Direct approximation of transfer function for the synthesis of recursive digital filters is presented. The equiripple approximation of the constant group delay is achieved.

To provide equal ripple delay response for IIR filters whose magnitude characteristics have n-th order zero at $z = -1$ or at the origin, it is formed the system of nonlinear equations and used iterative procedure to solve it. Numerical methods for solving the system of nonlinear equations are discussed. Finaly, cascade realization is given as second-order section form. It is investigated the optimal digital word length for the second-order section coefficients rounding, in order to accomplish this digital filter practical relization .

The group delay can be made constant with maximum error $\varepsilon_{\max}$ over proposed frequency band, up to full band $0 \le \theta \le \pi$. It can be achieved by τ_d value decreasing.

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