Invited paper

DYNAMIC SIMULATIONS IN ELECTRICAL APPARATUS

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Abstract: The paper gives an overview of the models for simulation of dynamics in electrical apparatus. The real problem most often consists of electromagnetic field, electric circuit and meahcnical motion problems. Three types of models are considered – circuit models, coupled models and decoupled models. Circuit models are based on magnetic circuit modelling, in coupled models the electromagnetic field is analysed together with the other two problems, while in decoupled model the field is analysed separately. Methods for the solution of the problems are outlined. Examples of different solved problems are given.

Keywords: Electrical apparatus, dynamic simulation, electromagnetic field, coupled problems.

INTRODUCTION

Dynamic simulations play important role in the design and study of a series of electrical apparatus, such as contactors, actuators, relays, circuit breakers, electromagnetic vibrators, etc.

For apparatus with two stable positions of the moving part dynamics of opening and closing is of interest. In this case the duration of the process to be simulated is short, most often of order of milliseconds. For apparatus with continuous motion, though, the dynamics determines the normal regime of the apparatus and their performance. For this case the duration depends on the period of the mechanical oscillations and the simulation should cover several periods.

Dynamics is of permanent interest to researchers and is of increased interest in recent years [1]-[31].

In the present paper, different models for simulation of dynamics in electrical apparatus will be presented, together with methods for solution to the problem and example of practical applications.

TYPES OF DYNAMIC MODELS

The real problem for solution of dynamics in electrical apparatus most often consists of three types of problems:

- Electromagnetic field problem
- Electric circuit problem
- Mechanical motion problem

In some cases, in addition to the three above mentioned problems, stress analysis and/or fluid dynamics problem can describe the real apparatus behaviour.

Depending on level of modelling of the different problems and on the coupling between them, we can distinguish three typical approaches to the real problem solution:

- Circuit approach
- Coupled approach
- Decoupled approach

Each approach involves its corresponding mathematical model. The essence of each approach lies in the model used for the electromagnetic field and in the way of coupling. Usually electromagnetic field is analysed using the finite element method as it is very well suited to solution of coupled problems of this type.

CIRCUIT MODELS

Circuit models are the oldest ones. They include magnetic circuit model (also called reluctance network model) instead of electromagnetic field model. This leads to the following three types of equations included in the circuit model:

- Magnetic circuit equations
- Electric circuit equations
- Mechanical motion equations

Most often the electrical apparatus are voltage supplied, which means that there is one electric circuit equation in the model.

Circuit models can be used at a preliminary stage of a device design as their accuracy is not very high. The most important for the level of circuit model as a whole is the level of the magnetic circuit model.

COUPLED APPROACH

Coupled models are the most closed to the real apparatus behaviour.

Mathematical model

The mathematical model of the coupled approach consists of the three types of equations

- Electromagnetic field equation
- Electric circuit equations
- Mechanical motion equations

The coupled model will be considered in more detail for the case of axisymmetrical electromagnetic field in cylindrical coordinate system (r,φ,z). This field is widespread in a variety of actuators. In general there can also be permanent magnets. If the motion is considered in one direction (z), the governing equation of the electromagnetic field becomes

$$\nabla \times \left(\nu \nabla \times \mathbf{A}\right) - \mathbf{J}_e + \sigma \frac{\partial \mathbf{A}}{\partial t} - \sigma \mathbf{v} \times \mathbf{B} - \nabla \times \left(\nu \mu_0 \mathbf{M}\right) = 0 , \quad (1)$$

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where

A is the magnetic vector potential having only one nonzero component (A_φ);

ν is the reluctivity;

$\mathbf{J}_e$ is the current density of the external sources (i.e. in the coil);

σ is the electrical conductivity;

t is time;

v is the velocity of the moving part;

B is the flux density;

μ_0 is the magnetic permeability of free space;

M is the magnetisation vector.

The last term of (1) takes into account the presence of permanent magnets.

If there is one coil and it is voltage supplied, the electric circuit equation is

$$u = Ri + \frac{d\Psi}{dt}, \tag{2}$$

where

u is the supply voltage;

R is the resistance of the coil;

Ψ is the flux linkages of the coil;

i is the current in the coil.

The mechanical motion equation is formed on the basis of the balance of the forces acting on the moving part

$$m\frac{d^2\zeta}{dt^2} + \beta\frac{d\zeta}{dt} = F_{em} - F_{load}, \tag{3}$$

where

m is the mass of the moving part;

ζ is the displacement of the moving part;

β is damping coefficient;

F_{em} is the electromagnetic force;

F_{load} is the load force.

In general, the load force can be of different kind, e.g., weight, spring, hydraulic or pneumatic force. In case of more complicated mechanical load, equations describing the load behaviour could be added.

All these three equations are to be presented in a form suitable for coupled solving. Eqn. (1) is first considered. After ignoring the velocity term, taking into account $\mathbf{B} = \nabla \times \mathbf{A}$ and carrying out vector operations in cylindrical coordinate system, the following equation is obtained (vector notation is omitted as all resulting vectors have only φ-components)

$$\frac{\partial}{\partial r}\left(\frac{\nu}{r}\frac{\partial}{\partial r}(rA)\right) + \frac{\partial}{\partial z}\left(\nu\frac{\partial A}{\partial z}\right) + J_e - \sigma\frac{\partial A}{\partial t} + \nu\mu_0\left(\frac{\partial M_r}{\partial z} - \frac{\partial M_z}{\partial r}\right) = 0 \, . \tag{4}$$

After introducing modified vector potential $A^\Psi = rA$ and expressing the current density in terms of the coil current, Eqn. (4) could be presented in the form

$$\frac{\partial}{\partial r}\left(\frac{\nu}{r}\frac{\partial A^\Psi}{\partial r}\right) + \frac{\partial}{\partial z}\left(\frac{\nu}{r}\frac{\partial A^\Psi}{\partial z}\right) + \eta_j\frac{\mathrm{N_c} i}{S_c} - \frac{\sigma}{r}\frac{\partial A^\Psi}{\partial t} + \nu\mu_0\left(\frac{\partial M_r}{\partial z} - \frac{\partial M_z}{\partial r}\right) = 0, \tag{5}$$

where $\mathrm{N_c}$ is the number of turns of the coil;

S_c is the c.s.a. of the coil;

η_j is parameter for correspondence between the point the equation is applied to and the cross section of the coil:

$$\eta_j = \begin{cases} 1 & \text{if the point belongs to the coil cross section;} \\ 0 & \text{otherwise.} \end{cases}$$

The flux linkage can be written using the modified vector potential. Then Eqn. (2) becomes

$$u - Ri - \frac{2\pi \mathrm{N_c}}{S_c}\int_{S_c}\frac{\partial A^\Psi}{\partial t}dS_c = 0 \, . \tag{6}$$

To reduce the order of derivatives in the force equation (3), the latter is split into two equations:

$$m\frac{d\mathrm{v}}{dt} + \beta\mathrm{v} - F_{em} + F_{load} = 0 \tag{7}$$

$$\frac{d\zeta}{dt} - \mathrm{v} = 0 \tag{8}$$

The system of equations to be solved is (5)-(8), unknown functions are A^Ψ, i, v and ζ. This system should be solved under the corresponding initial and boundary conditions.

Approach implementation

As stated before, the main points of the approach are:

- Method for electromagnetic field computation;
- Method for integration in time;
- Coupling approach.

Finite element method will be considered for electromagnetic field analysis. An important point is the motion modelling in finite element analysis. For this purpose, several basic approaches are used so far:

- *Lockstep mesh.* The moving part is allowed to move only through fixed steps;

- *Lagrange multipliers*. This scheme involves the use of a general slip surface between two disconnected meshes. The continuity of the field components is ensured using Lagrange multipliers;
- *Mesh re-generation*. This is a simple and sometimes effective method. It employs re-meshing at each time step. Potential mesh quality problems can occur;
- *Boundary element methods*. They are applied to model the slip region. This produces dense submatrices from all the nodes on the slip surface. Similar methods are those based on macroelement representation of the slip region;
- *Finite elements with incomplete shape functions*. They are very useful when the slip region is of constant width.
- *Overlapping meshes*. Two independent meshes – one for the stationary and one for the moving part are used, and overlapping is allowed.

The variety of approaches is a sign that there is not still universal approach, applicable in all cases. That is why for each particular problem suitable approach is preferred. The majority of the above mentioned approaches overcome the problem with the velocity term in field equation. If this term is considered, the system matrix becomes nonsymmetric due to the presence of the velocity term. In this case, application of Galerkin method is possible only when the cell Peclet number is less than 2. Otherwise, upwinding techniques should be employed. They, though, are applicable to fixed meshes, which limits their application. That is why the majority of the researchers employ approaches that keep the mesh in the moving part unchanged with respect to a coordinate system fixed to the moving part. In this case, the velocity term is omitted and the finite element system matrix becomes symmetric.

For integration in time, numerical method for solving ordinary differential equations should be employed. Usually for coupled models low-order methods are used, as the number of equations becomes large – of typical order from thousands and dozens of thousands for two-dimensional problems to hundreds of thousands for three-dimensional cases. Here, Backward Euler method is presented for integration in time of the whole system of equations.

If k and $k+1$ denote two successive moments, e.g. t and $t+\Delta t$, the system of equations becomes

$$\frac{\partial}{\partial r}\left(\frac{\nu}{r}\frac{\partial A^{\psi}_{k+1}}{\partial r}\right)+\frac{\partial}{\partial z}\left(\frac{\nu}{r}\frac{\partial A^{\psi}_{k+1}}{\partial z}\right)+\eta_j\frac{\mathrm{N_c} i_{k+1}}{S_c}-\frac{\sigma}{r\Delta t}A^{\psi}_{k+1} + \nu\mu_0\left(\frac{\partial M_r}{\partial z}-\frac{\partial M_z}{\partial r}\right)_{k+1}+\frac{\sigma}{r\Delta t}A^{\psi}_{k}=0 \quad (9)$$

$$u_{k+1}-Ri_{k+1}-\frac{2\pi \mathrm{N_c}}{S_c}\int_{S_c}\frac{A^{\psi}_{k+1}-A^{\psi}_{k}}{\Delta t}dS_c=0 \quad (10)$$

$$m\frac{\mathrm{v_{k+1}}}{\Delta t}+\beta \mathrm{v_{k+1}}-F_{em\,k+1}+F_{load\,k+1}-m\frac{\mathrm{v_k}}{\Delta t}=0 \quad (11)$$

$$\frac{\zeta_{k+1}}{\Delta t}-\mathrm{v_{k+1}}-\frac{\zeta_k}{\Delta t}=0 \quad (12)$$

Let us consider finite element method using first order triangular elements to be applied for solving the field equation. The modified vector potential is defined by the vector of shape functions [N] and the vector of modified potential values at the nodes [A^{ψ}]. Galerkin variant of the method is employed.

The field equation can be presented in the form

$$\mathbf{f}^{\mathrm{a}}\left(\mathbf{a}_{k+1}, i_{k+1}, \zeta_{k+1}\right)=\mathbf{0}, \quad (13)$$

where $\mathbf{a}_{k+1}$ is the vector of values of the potential A^{ψ} (of dimension Q_n, where Q_n is the total number of nodes in the FE mesh) at the $k+1$-st time step, i.e. $\mathbf{a}_{k+1} = \left[A^{\psi}\right]_{k+1}$. The dependence on ζ_{k+1} is included in the shape functions and their derivatives.

The circuit equation can be written as

$$f^{\,i}\left(\mathbf{a}_{k+1}, i_{k+1}\right)=0 \quad (14)$$

Attention should also be drawn to the force equation. It has the electromagnetic force acting on the moving part as a term. The force is usually calculated either by Maxwell stress tensor of by virtual work approach. For linear finite elements, if shape functions are used for flux density or magnetic energy evaluation, its implementation into computer code is relatively easy. However, it is well known that the accuracy of an approach based on shape functions for linear elements is not very high. This could lead to significant errors when dynamics is being simulated.

For improving the accuracy of force calculation averaging the flux density at nodes is preferred. This approach increases the order of approximation of the flux density – instead of constant, as it is in the shape functions approach, it becomes a linear function of the coordinates. This is an approach often used at post-processing level, when values of the magnetic vector potential are already known. Here, Maxwell stress tensor approach is presented for electromagnetic force evaluation and for improving the accuracy several integration surfaces are taken.

As the motion is in z-direction only, the electromagnetic force is defined by the flux density components B_r and B_z as

$$F_{em}=\frac{1}{\mu_0 p}\sum_{q=1}^{p}\oint_{S_{pq}}\begin{bmatrix}B_r B_z\cos\left(\vec{n}_q,\vec{e}_r\right)+\\ \frac{1}{2}\left(B_z^2-B_r^2\right)\cos\left(\vec{n}_q,\vec{e}_z\right)\end{bmatrix}dS_{pq}, \quad (15)$$

where p is the number of integration surfaces, each of them denoted as S_{pq};

$\vec{n}_q$ is outward normal unit vector to the surface S_{pq};

$\vec{e}_r$, $\vec{e}_z$ are unit vectors in *r*- and *z*-direction.

Eqns. (11) and (12) could be written in a form similar to that of (13) and (14) as follows:

$$f^{\,\mathrm{v}}\left(\mathbf{a}_{k+1}, \mathrm{v}_{k+1}\right) = 0\,, \tag{16}$$

$$f^{\,z}\left(\mathrm{v}_{k+1}, \zeta_{k+1}\right) = 0\,. \tag{17}$$

The resulting system (13), (14), (16) and (17) is non-linear.

Two groups of coupling approaches are possible: strong or weak coupling.

Strong coupling itself can be realised through two approaches: direct and indirect coupling. The direct coupling involves simultaneous solution of all equations. In order to realise it, though, the electromagnetic force should be presented in terms of the magnetic vector potential and in general it is second-order relationship. This introduces additional non-linearity in the system and that is why this approach is not often used. Indirect strong coupling involves indirect coupling of (13)-(14) and (16)-(17) in a iterative way to realise the implicit procedure at each time step. Here, there are several iterations of the solutions of the couples of equations at each time step.

Weak coupling consists of sequential solution of (13)-(14) and (16)-(17) for each time step. Here, only one iteration per time step is carried out. Weak coupling is not recommended as depending on the particular problem it can lead to significant errors.

The main advantages of the coupled model are connected with its high accuracy and ability to simulate the real problem in most adequate way. Its main drawback is the amount of computer resources, especially for three-dimensional problems and for multiply changed conditions.

Practical examples

Three examples will be given. The first two are of solved problems for simulation of dynamics using strong coupled indirect approach – solenoid actuator and reciprocating permanent magnet linear actuator. The third one is for modelling of the breaking process of SF6 circuit breaker with arc rotation.

The first example is a solenoid actuator shown in Fig. 1. It is an example of a device with two stable positions.

The actuator has been studied during the transient process of switching, when spring. The supplied voltage was 56 V, the resistance of the coil – 255 Ω, number of turns – 5800, initial air gap – 8.2 mm, final air gap – 2.5 mm. The spring force value at the initial air gap is 2.6 N; the spring coefficient – 670 N/m, the mass of the mover - 0.16 kg. Total number of FE nodes has varied between 5515 at the initial gap and 5053 at the final gap.

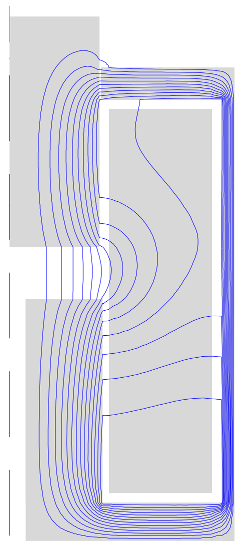

Fig.1 – *Solenoid actuator with static field plot*

The simulation results obtained for the variation in time of the coil current, electromagnetic force, displacement and velocity of the mover are shown in Fig. 2 – Fig. 5.

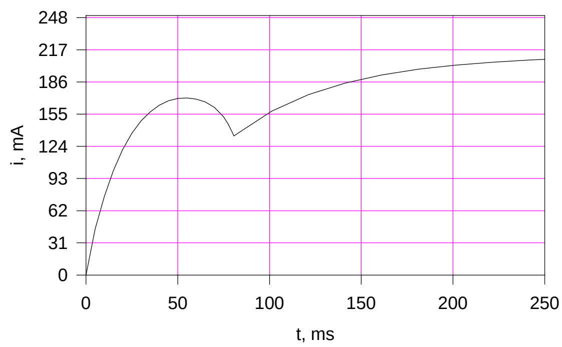

Fig.2 – *Solenoid actuator – coil current*

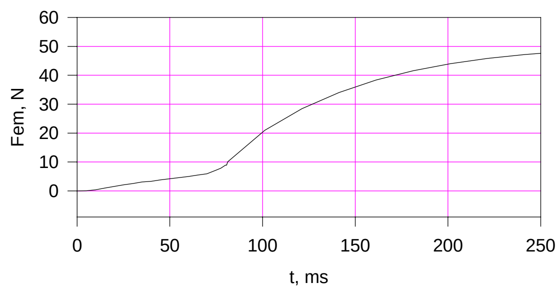

Fig.3 – *Solenoid actuator – electromagnetic force*

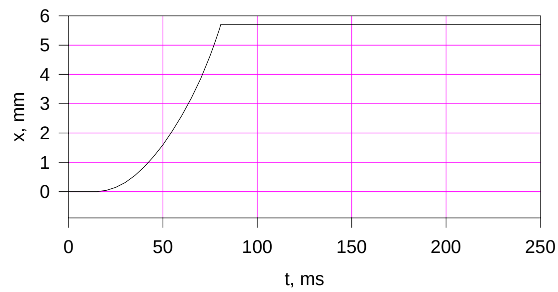

Fig.4 – *Solenoid actuator - displacement*

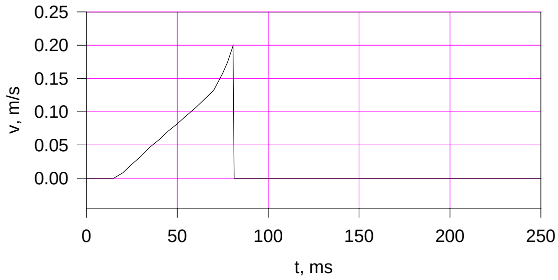

Fig.5 – *Solenoid actuator- velocity*

The coil current has been verified experimentally.

The second example is reciprocating permanent magnet linear actuator with moving permanent magnet (referred to hereafter as moving magnet actuator). It is an example of a device with continuous motion.

The actuator geometry is shown in Fig. 6.

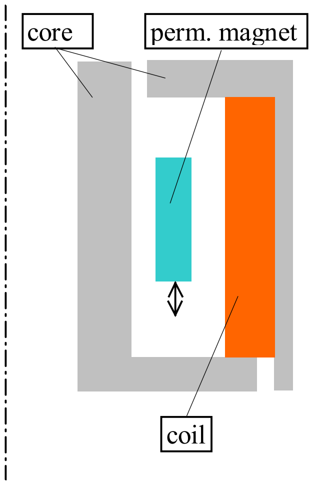

Fig.6 – *Principal construction of the moving magnet actuator*

Results were obtained for both static and dynamic case of a linear actuator with ferrite permanent magnet. The direction of magnetisation of the magnet is radial, the residual flux density being 0.37 T. The ferromagnetic core is massive and eddy currents there are taken into account. The number of turns of the coil is 5900. The maximal working stroke of the actuator is 10 mm (±5 mm from the symmetry position of the permanent magnet).

The number of finite elements used in simulations was about 9 000. A fragment of the finite element mesh near the lower part of the permanent magnet is shown in Fig. 7 where the Maxwell zone used for force computation can also be seen.

The actuator was simulated at sinusoidal supply voltage. The load force is a spring one of zero value at symmetry position of the mover (permanent magnet) and stiffness of 1 N/mm for both directions. In Fig. 5, time variations of the voltage, current, electromagnetic force, displacement and velocity of the mover are shown.

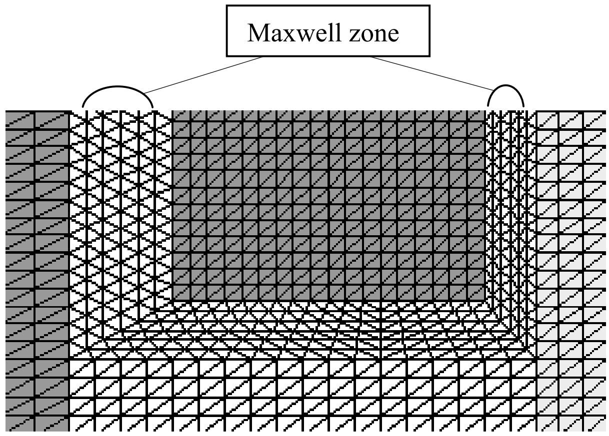

Fig.7 – *Fragment of the mesh near the lower part of the permanent magnet*

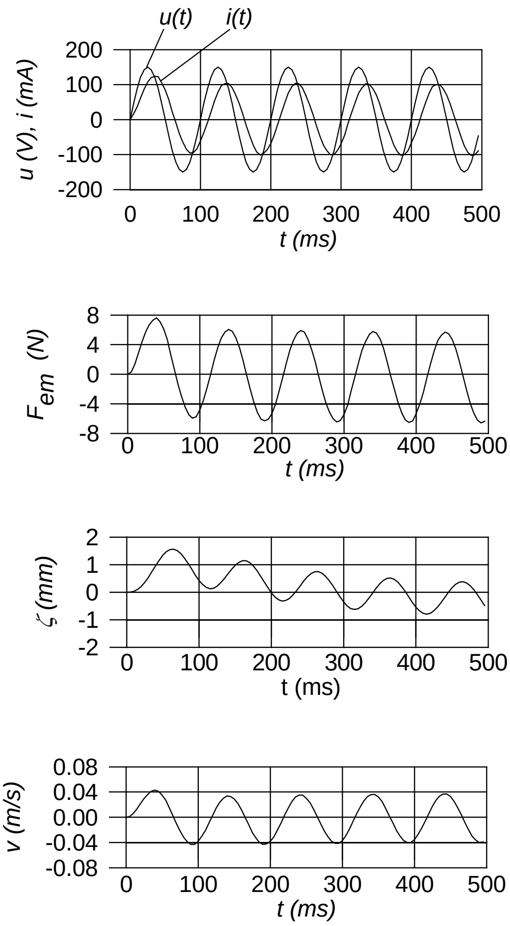

Fig.8 – *Moving magnet actuator - voltage* u, *current* i, *electromagnetic force* F_{em}, *displacement* ζ *and velocity* v *versus time* t *at 10 Hz frequency*

Due to the non-symmetry in axial direction, oscillations of the mover are not symmetrical around its symmetry position. A field plot for the dynamic case is shown in Fig. 9.

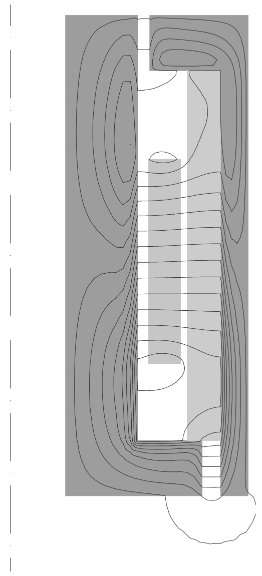

Fig.9 – *Moving magnet actuator - field plot at* t *= 435 ms*

The third example is connected with opening of SF6 circuit breaker with arc rotating in magnetic field. The construction of the circuit breaker is shown in Fig. 10.

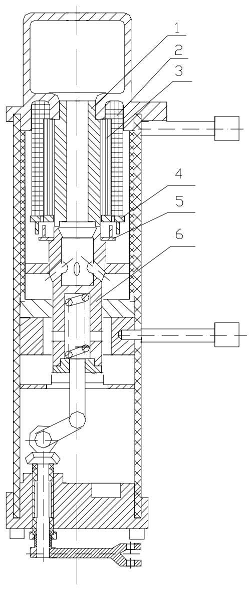

Fig.10 – ***Principal geometry of the studied circuit breaker:***

1 – ***fixed main contact;* 2** – ***coil;* 3** – ***magnetic core;* 4,5** – ***arcing rings;* 6** – ***moving main contact***

The dynamic process of current breaking requires solution to coupled problem involving electromagnetic field, mechanical motion due to the driving mechanism, fluid dynamics, electric arc motion and the processes in the arc itself. In the presented example, some simplifications are used in order to make the problem solvable for practical purposes. The coupled approach used involves electromagnetic field, mechanical motion of the moving contact and arc rotation due to its interaction with the magnetic field.

The magnetic field is governed by Eqn. (1) in cylindrical co-ordinate system. It is solved under initial and boundary conditions. The initial conditions are zero conditions for the magnetic vector potentials as magnetic field is initiated after the start of the interrupting process. The current flowing before interruption excites magnetic field only in azimuthal direction. The magnetic field problem is solved under homogeneous Dirichlet boundary conditions on the boundary of a buffer zone around the extinguishing chamber of the circuit breaker.

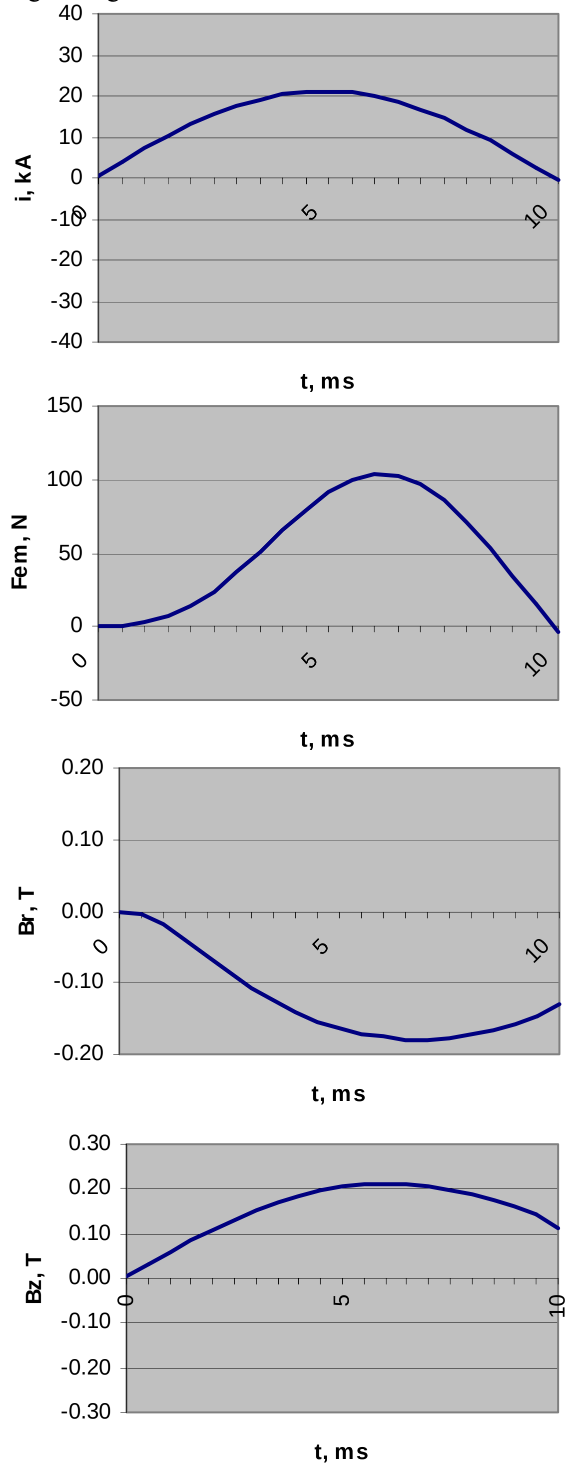

Fig. 11 - *Waveforms of the interrupting current, electromagnetic force acting on the arc and radial component of the flux density in the arc zone*

Time stepping and backward Euler method are used for solving the problem. The solution process starts from the moment of transferring the arc from the main to the arcing contacts. The time for this moment is obtained from the law of movement of the moving contact. At the same moment the current is transferred to the coil. The magnetic field analysis is carried out using the finite element method. As there is a movement, at each time step the mesh of the moving parts is moving and new elements introduced with the extension of the gap between contacts. The system of non-linear equations is solved using relaxation method with underrelaxation with respect to the magnetic permeability. The electromagnetic force acting on the arc is determined by the Laplace's law.

The results are obtained for 15 kA rms value of sinusoidal interrupting current. The same approach can be used for other waveforms of the current. In Fig. 11, the results for the flux density and the electromagnetic force acting on the arc are shown. The eddy currents in the conductive parts cause a phase shift between the current and the flux density components and a decrease of the magnitude of the electromagnetic force. Due to the movement of the moving contact the magnitudes of the components of the average flux density in the arc zone descend while in the case of fixed contacts they remain the same.

DECOUPLED APPROACH

The essence of the decoupled approach is to separate solution to the electromagnetic field equation from the rest of the problem. All necessary parameters in the other equations that are determined by the field, are obtained using some kind of approximation of the field results.

Mathematical model

For the decoupled model, instead of Eqn. (1), the field is described by Poisson equation with respect to the magnetic vector potential. The other equations remain the same. In this case, it is necessary to modify the electric circuit equation in a way suitable for utilization of the field results or for better presentation of the dynamic parameters of the device.

The presentation of the flux linkage and its derivative with respect to the time can be carried out in different ways, depending on the level of including the inductance L (defined by $L = \Psi/i$) in the flux linkage presentation. Thus the time derivative of the flux linkage can be one of the expressions below

$$\frac{d\Psi}{dt} = \frac{\partial\Psi}{\partial x}\frac{dx}{dt} + \frac{\partial\Psi}{\partial i}\frac{di}{dt} \tag{18}$$

$$\frac{d\Psi}{dt} = \frac{\partial(Li)}{\partial x}\frac{dx}{dt} + \frac{\partial(Li)}{\partial i}\frac{di}{dt} = i\frac{\partial L}{\partial x}\frac{\partial x}{\partial t} + L\frac{di}{dt} + i\frac{\partial L}{\partial i}\frac{di}{dt} \tag{19}$$

For some cases it is possible to use mixed expression, i.e. both flux linkage and inductance derivatives with respect to the displacement x and the current i.

The three most used ways of presentation of the electric circuit equation in normal form are

$$\frac{di}{dt} = \frac{1}{\dfrac{\partial\Psi}{\partial i}}\left(U - Ri - \frac{\partial\Psi}{\partial x}\mathrm{v}\right) \tag{20}$$

$$\frac{di}{dt} = \frac{1}{L + i\dfrac{\partial L}{\partial i}}\left(U - Ri - \frac{\partial\Psi}{\partial x}\mathrm{v}\right) \tag{21}$$

$$\frac{di}{dt} = \frac{1}{L + i\dfrac{\partial L}{\partial i}}\left(U - Ri - i\frac{\partial L}{\partial x}\mathrm{v}\right) \tag{22}$$

Eqns. (20) and (21) are suitable to apparatus with permanent magnets, while (22) is suitable to devices without permanent magnet.

The rest two equations are also presented in normal form

$$\frac{dx}{dt} = \mathrm{v} \tag{23}$$

$$\frac{d\mathrm{v}}{dt} = \frac{1}{m}\left(F_{em} - \beta\mathrm{v} - F_{load}\right) \tag{24}$$

Thus the mathematical model for the decoupled approach consists of one of the equations (20), (21), (22) and (23)-(24).

All unknown functions are obtained from the magnetic field analysis.

In case of additional circuit elements or complex mechanical loads, the corresponding governing equations can be added easily.

Approach implementation

Two types of methods underlie the approach implementation:

- Method for the magnetic field analysis
- Method for the solution of the system of ordinary differential equations

Another important point in the approach implementation is the way of obtaining the necessary functions from the field analysis.

Usually the finite element method is used for magnetic field analysis. For the solution of the system of ordinary differential equations variety of methods can be used. Here it is better to use higher order method as the system consists of only several equations, most often 3.

Let us consider a model with Eqn (20). The necessary functions to be obtained from field analysis are $\frac{\partial\Psi}{\partial x}(x,i)$, $\frac{\partial\Psi}{\partial i}(x,i)$ and $F_{em}(x,i)$. If the model includes (22) instead of (20), then the necessary functions will be $L(x,i)$, $\frac{\partial L}{\partial x}(x,i)$, $\frac{\partial L}{\partial i}(x,i)$ and $F_{em}(x,i)$.

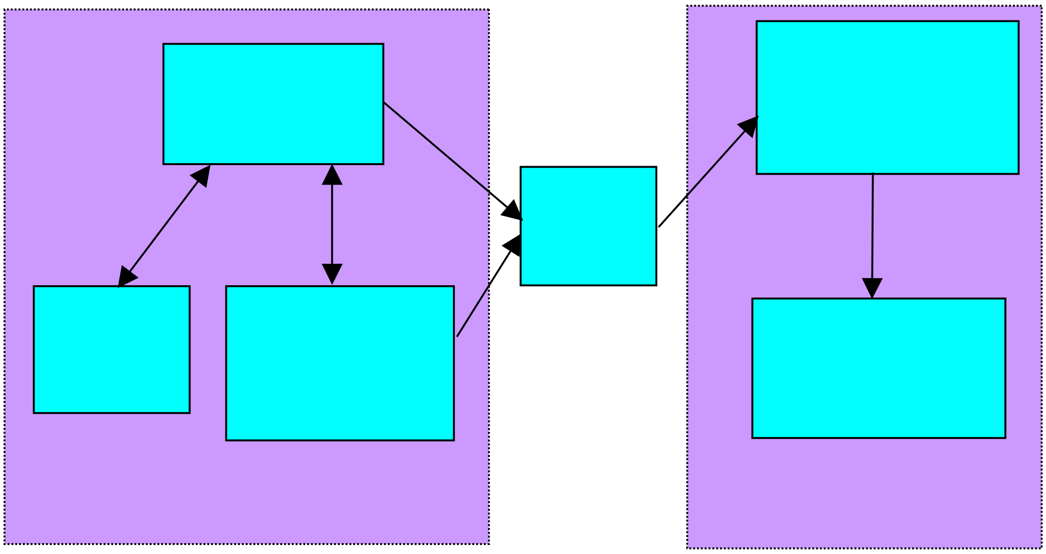

***Fig.12** –Implementation of the decoupled approach*

An example of practical implementation of the approach for the system of equations (21),(23),(24), axisymmetric magnetic field and presence of permanent magnet is shown in Fig. 12. This implementation involves bicubic spline approximation of the field results, which allows easy obtaining the necessary functions and their derivatives. Similar implementations can be realized for the other two systems of equations (with (20) or (22) instead of (21)), as well as for three-dimensional problems. In this case the program for FEM analysis should be three-dimensional.

The program FEMM [16] can be used for finite element analysis in case of 2D or axisymmetric fields. In order to obtain the desired function both current and displacement should be varied. A grid displacement-current is defined and at each point of this grid two magnetic field analyses are performed. The first one is with all real data. From this analysis the total flux linkage and the electromagnetic force are obtained. In the second analysis the permanent magnet is replaced with a ferromagnetic body of the same magnetic permeability, i.e. the coercive force of the magnet is set to zero. From this analysis the result for the coil inductance is obtained. All these results are stored in an intermediate data file. For automatic generating of the coil-displacement grid and obtaining the desired results, a program is written in Lua scripting language.

The rest of the analysis is carried out using the Matlab® package [14]. First, the data from the intermediate file are used for creating bicubic spline approximations of the functions Ψ (x,i), L(x,i) and F_{em}(x,i). After that, the set of ordinary differential equations is solved numerically. The necessary derivatives and are obtained from the bicubic spline approximations.

The advantages of the decoupled approach is that it allows fast multiple solution for different supply voltages, external circuit parameters or mechanical loads. Once the bicubic spline approximation is obtained, all these changes can be taken into account in the last part - solving the system of ordinary differential equations. The approach allows fast multiple solutions for different supply voltages. In case of change the supply voltage or some external circuit parameters, only the Matlab solution of the system of ordinary differential equations is to be performed. The drawback is the neglecting of eddy currents.

Practical examples

Three examples will be presented. The first one is a bistable linear actuator with moving permanent magnet (called for brevity bistable actuator – Fig. 13). The second one is the same moving magnet actuator presented in the coupled approach. The third one is an example for a contactor with 3D field analysis.

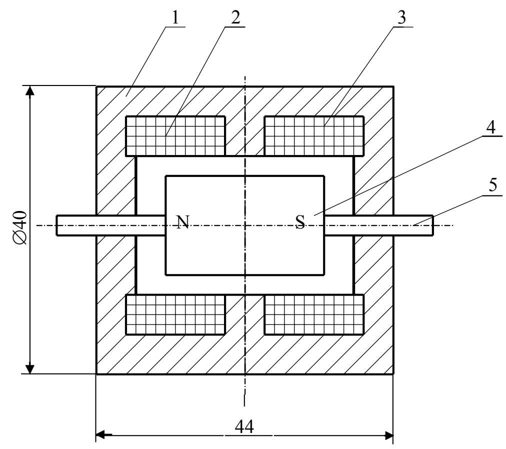

Fig.13 – *Bistable actuator*

The actuator for the first example consists of a core 1, two exciting coils 2,3 and moving permanent magnet 4 fixed on non-magnetic shaft 5. The permanent magnet is magnetised in axial direction. The two coils are connected in series in a way to create opposite m.m.f.-s. As a result axial electromagnetic force arises on the permanent magnet. The direction of the force depends on the direction of the current. The mover is held at the end positions by the permanent magnet, without current in the coils.

The results are obtained for the following data: supply voltage U = 24 V DC; coil resistance R = 12 Ω; number of turns: 700; mass of the mover *m* = 32 g. The axially magnetised permanent magnet (Ba ferrite, H_c = 104 kA/m, B_r = 0.17 T) is of dimensions ∅20/∅5×15 mm.

The actuator is analysed in vertical position. Two groups of results are obtained – one for motion up and one for motion down. In fig. 14 the results for current, displacement, velocity and electromagnetic force are shown for motion up.

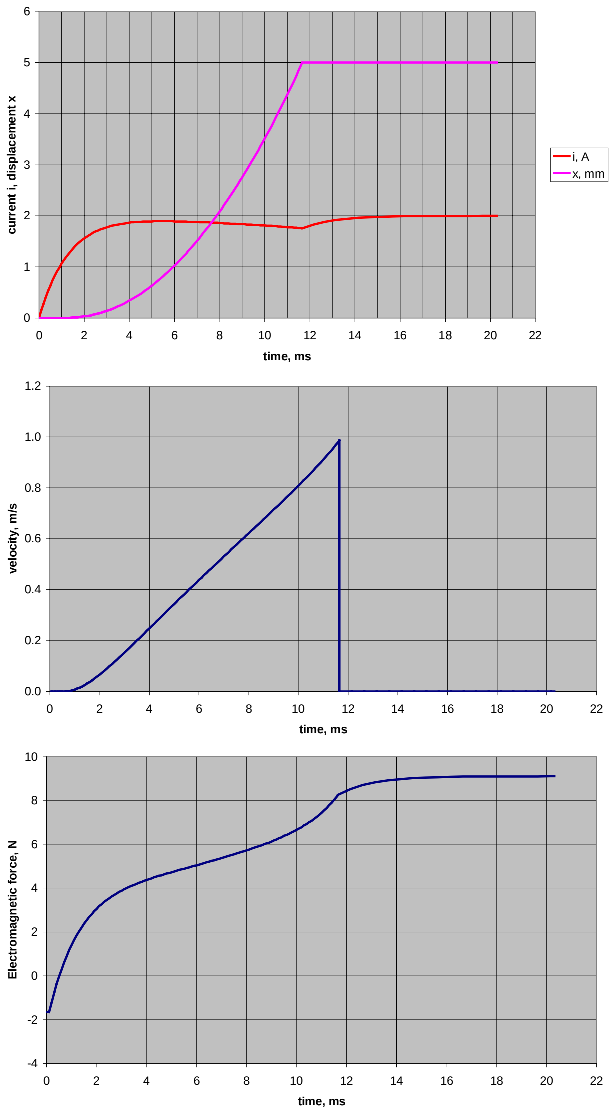

Fig.14 – *Bistable actuator- results*

The results for the second example – the moving magnet actuator – are given in Fig. 15 together with the ones obtained using the coupled approach.

As seen, the difference between the two approaches is not very large. This means that for lower frequencies the decoupled approach can give satisfactory accuracy.

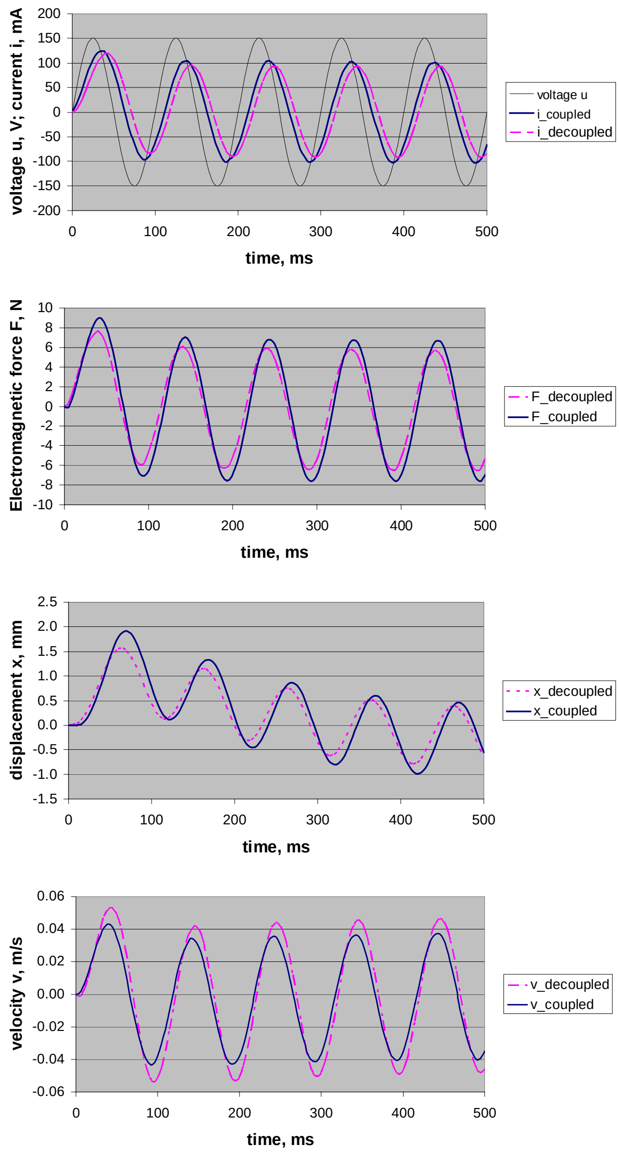

Fig.15 – *Moving magnet actuator – results using coupled and decoupled approach*

The third example is a DC contactor. The driving electromagnet is shown in Fig. 16. In this case, three-dimensional field computation is carried out using ANSYS® program [2].

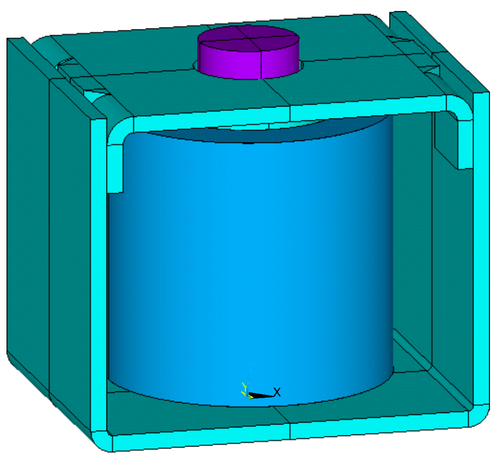

Fig.16 – *Driving electromagnet of DC contactor*

The results are obtained for the following data: supply voltage U = 55 V DC; coil resistance R = 222 Ω; number of turns: 5900; mass of the mover m = 158 g. The stiffness of the spring is 370 N/m, the initial spring force – 2.6 N.

As for all cases of decoupled approach, attention should be drawn to the algorithm ruling the solution of the system of ordinary differential equation in order to achieve exact enough final position of the mover.

The results for this example are shown in Figs. 17-20.

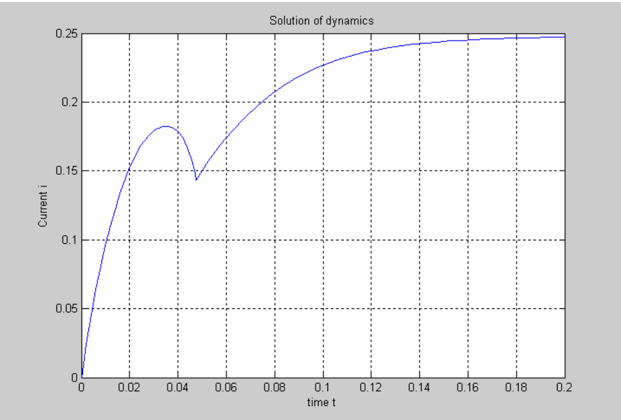

Fig.17 – *DC contactor- coil current*

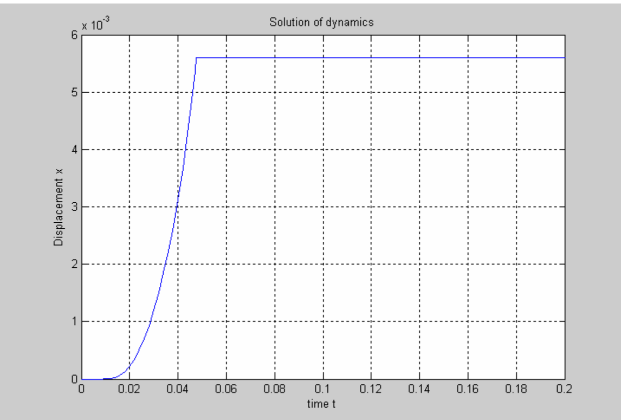

Fig.18 – *DC contactor- displacement*

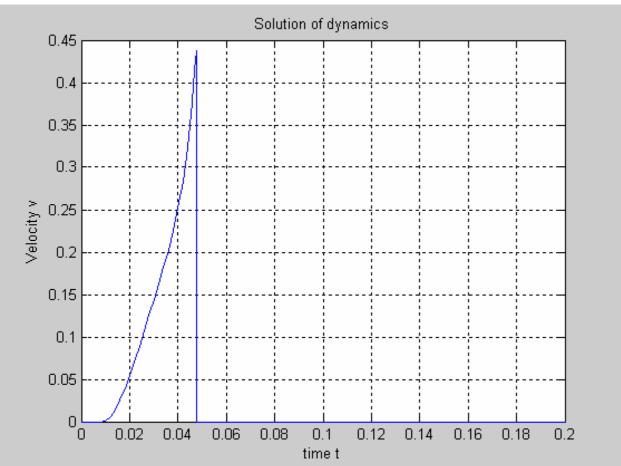

Fig.19 – *DC contactor- velocity*

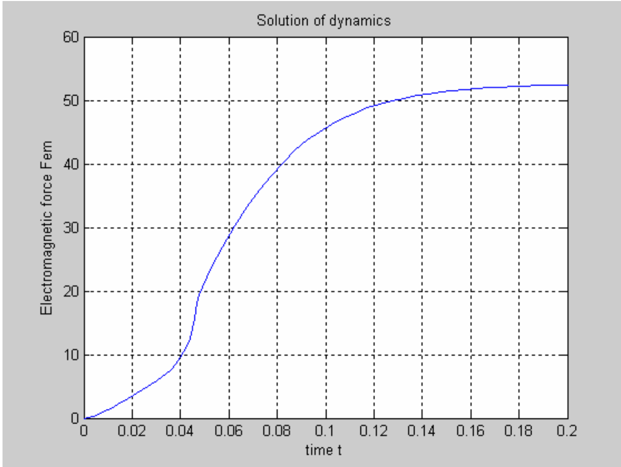

Fig.20 – *DC contactor- coil inductance*

CONCLUSION

Typical approaches for dynamic simulations in electrical apparatus were presented.

The circuit approach depends on the level of the reluctance network model.

The coupled approach is the most accurate one and the closest to the real device behaviour. Its main drawback is the large amount of computational resource, especially in three-dimensional problems and multiple simulations.

The decoupled approach with bicubic spline approximations of the magnetic field analysis results can be more flexible than the coupled model for study of the apparatus behaviour at different external conditions, e.g., different circuit parameters and mechanical parameters of the load mechanism. It is also suitable for modelling of the apparatus behaviour in a larger system. Coupling with additional dynamics equations can be easily implemented. It has the drawback of neglecting the eddy currents and therefore should be used with care for fast processes and presence of solid conductors.

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