CYMOMOTIVE FORCE OF THE VERTICAL DIPOLE ANTENNA

Milica RANČIĆ[1]

Abstract: The Cymomotive force (CMF) of a vertical dipole antenna (VDA) placed above a lossy half-space, is determined in this paper. The unknown current distribution was determined solving the system of integral equations of Hallen's type (SIE-H) using the point-matching method and assuming polynomial current approximation. The Sommerfeld's integral kernel (SIK) was approximated using a simple, accurate and general model.

Keywords: Cymomotive force, Vertical dipole antenna, Sommerfeld's integral

INTRODUCTION

During the first Regional administrate conference for medium and long wave propagation held in Geneva in 1974, a new quantity - Cymomotive force (CMF) was introduced as basic characteristics of MW and LW aerials. The cymomotive force is, in certain direction, defined as product of electrical field intensity originating from an antenna, and distance between the emitting aerial and the observed field point, ([2], [3]). In this paper, the CMF of a vertical dipole antenna (VDA) placed above a lossy half-space is determined. The semi-conducting ground is treated as linear, homogenous and isotropic medium of known electrical parameters.

For this purpose, the system of integral equations of Hallen's type (SIE-H) was solved applying the moment method (MoM), assuming the polynomial approximation for the unknown current distribution (UCD) along the antenna conductors. The influence of finite ground conductivity on VDA characteristics, expressed by the Sommerfeld's integral kernel (SIK), was taken into account. In order to solve this complex type of integrals that occur in the expressions for the Hertz's vector and electrical scalar potential, a simple model for the SIK was applied. After the UCD is obtained, the radiation field and the CMF are easily determined.

Numerical results confirming the validity of the applied SIK model, as well as those for the CMF of the considered VDA, are given in the last section of this paper.

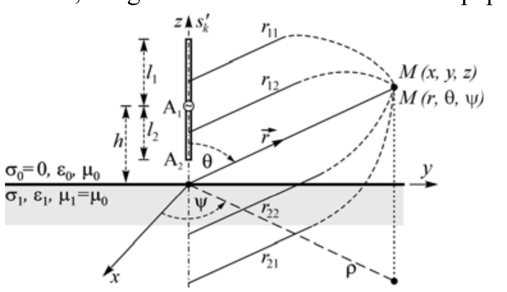

Fig.1 - *The illustration of the VDA above a lossy half-space with the far field point M.*

THEORETICAL BACKGROUND

Problem layout

The VDA placed above a homogenous and isotropic lossy half-space, fed by an ideal voltage δ - generator, of voltage $U=1\text{V}$ and angular frequency $\omega=2\pi f$, is considered, Fig.1. The antenna conductors are of length l_k, $k=1,2$ and circular cross-section of radius a_k, $a_k << l_k$ and $a_k << \lambda_0$ (λ_0 - wave length in air). The air and ground parameters are: $\sigma_0=0$, ε_0, μ_0, σ_1, $\varepsilon_1=\varepsilon_{r1}\varepsilon_0$, $\mu_1=\mu_0$. The following labels were also introduced: $\underline{\gamma}_i=(j\omega\mu_i\underline{\sigma}_i)^{1/2}$, $i=0,1$ - complex propagation constant, $\underline{\sigma}_i=\sigma_i+j\,\omega\,\varepsilon_i=j\omega\varepsilon_0\underline{\varepsilon}_{ri}$, $i=0,1$ - complex conductivity, $\underline{n}=\underline{\gamma}_1/\underline{\gamma}_0=\underline{\varepsilon}_{r1}^{1/2}$ - refraction index and $\underline{\varepsilon}_{r1}=\varepsilon_{r1}-j\varepsilon_{i1}=\varepsilon_{r1}-j60\sigma_1\lambda_0$ - complex relative permittivity. The UCD localised along the conductor axis is denoted by $I_k(s'_k)$, $0\le s'_k\le l_k$, $k=1,2$. Beginnings of s'_k - axes are at points $z_{A1}=h$ and $z_{A2}=h-l_2$ (i.e. $z'_k=z_{Ak}+s'_k$, $k=1,2$). The height of the antenna feed point is h, $h\ge l_2$.

Evaluation of the UCD

In order to determine the UCD along the antenna conductors, the system of integral equations of Hallen's type was solved. The SIE-H has the following implicit form:

$$\Pi_{z0}\left(s'_k\right)=+\pi_{ek}\ ch\,(\underline{\gamma}_0 s'_k)-\frac{V_{ek}}{\underline{\gamma}_0}\ sh\,(\underline{\gamma}_0 s'_k)\,, \qquad (1a)$$

and

$$\varphi_0\left(s'_k\right)=-\underline{\gamma}_0\ \pi_{ek}\ sh\,(\underline{\gamma}_0 s'_k)+V_{ek}\ ch\,(\underline{\gamma}_0 s'_k)\,, \qquad (1b)$$

where: $\pi_{ek}, k=1,2$ - the unknown integration constants (the Hertz's vector potential at the beginning of the k$^{\text{th}}$ conductor) and $V_{ek}, k=1,2$ - the electrical scalar potential at the beginning of the k$^{\text{th}}$ conductor. $\Pi_{z0}\left(s'_k\right)$ and $\varphi_0\left(s'_k\right)$ are the Hertz's vector and the electrical scalar potential, respectively, calculated at the arbitrary point on the surface of the k$^{\text{th}}$ conductor, $0\le s'_k\le l_k$, $k=1,2$, and are given by:

$$\Pi_{z0}=\frac{1}{4\pi\underline{\sigma}_0}\sum_{k=1}^{2}\int_{s'_k=0}^{l_k} I_k(s'_k)[K_0(r_{1k})+S_{00}^{v}(r_{2k})]\,\mathrm{d}s'_k\,, \quad (2)$$

[1] University of Niš, Department of Theoretical Electrical Engineering, Aleksandra Medvedeva 14, 18000 Niš, Serbia, E-mail: milica@elfak.ni.ac.yu

$$\varphi_0 = \frac{1}{4\pi \underline{\sigma}_0} \sum_{k=1}^{2} \int_{s_k'=0}^{l_k} I_k(s_k') \frac{\partial}{\partial s_k'} [K_0(r_{1k}) - S_{00}^v(r_{2k})] \mathrm{d} s_k' \,. \quad (3)$$

The unknown current distribution (UCD), denoted by $I_k(s_k')$ in previous two expressions, is assumed in a polynomial form as follows:

$$I_k(s_k') = \sum_{m=0}^{M_k} I_{km} (s_k' / l_k)^m, 0 \le s_k' \le l_k, k = 1,2, \quad (4)$$

where I_{km} are unknown current coefficients, and M_k the polynomial degree, [4].

The SIE-H can be numerically solved applying the point-matching method, having the matching points

$$s_{ki} = (l_k / M_k) i, i = 0,1,2,\ldots, M_k, k = 1,2 \,, \quad (5)$$

and fulfilling the following boundary conditions

$$I_1(l_1^-) = 0, I_2(0^+) = 0, I_2(l_2^-) = I_1(0^+), \quad (6)$$

$$-\varphi_0(s_2 = l_2^-) - U + \varphi_0(s_1 = 0^+) = 0. \quad (7)$$

In the expressions for the Hertz's vector and electrical scalar potential (Eqs. 2, 3), a certain type of integrals that describe the influence of the finite ground conductivity above which the VDA is placed, occur. In literature they are referred to as integrals of Sommerfeld's type, and are defined by

$$S_{00}^v(r_{2k}) = \int_{\alpha=0}^{\infty} \tilde{R}_{z10}(\alpha) \cdot \tilde{K}_0(\alpha, r_{2k}) \mathrm{d}\alpha \,, \quad (8)$$

where $\tilde{R}_{z10}(\alpha)$ and $\tilde{K}_0(\alpha, r_{2k})$ - spectral reflection ceofficient (SRC) and standard potential kernel from the image in the flat mirror, respectively. They are defined as:

$$\tilde{R}_{z10}(\alpha) = \tilde{R}_{z10}(u_0) = \frac{\underline{\varepsilon}_{r1} u_0 - u_1}{\underline{\varepsilon}_{r1} u_0 + u_1} \,, \quad (9)$$

$$\tilde{K}_0(\alpha, r_{2k}) = \frac{\mathrm{e}^{-u_0(z+z_k')}}{u_0} \alpha J_0(\alpha\rho) \,, \quad (10a)$$

$$K_0(r_{2k}) = \frac{\mathrm{e}^{-\underline{\gamma}_0 r_{2k}}}{r_{2k}} = \int_{\alpha=0}^{\infty} \tilde{K}_0(\alpha, r_{2k}) \mathrm{d}\alpha \,, \quad (10b)$$

where: $u_0 = \sqrt{\alpha^2 + \underline{\gamma}_0^2}$, $u_1 = \sqrt{\alpha^2 + \underline{\gamma}_1^2}$, $J_0(\alpha\rho)$ - zero order of the first kind Bessel's function, and $r_{2k} = \sqrt{\rho^2 + (z + z_k')^2}$.

In order to solve the expression (8) in a closed form, a simple model for the SRC, proposed in [5], is applied. It is in a form of a rational function with two unknown constants:

$$\tilde{R}_{z10}(u_0) \cong B + A \frac{\underline{\gamma}_0}{u_0} \,. \quad (11)$$

Unknown constants B and A are determined by matching the expressions (9) and (11) at points $u_0 = \underline{\gamma}_0$ and $u_0 \to \infty$. This way, the following is obtained:

$$B = R_\infty = (\underline{n}^2 - 1)/(\underline{n}^2 + 1) \,, \quad (12a)$$

$$A = (R_0 - R_\infty) \,,\; R_0 = (\underline{n} - 1)/(\underline{n} + 1) \,. \quad (12b)$$

Substituting (11) into (8), the following approximate expression for the Sommerfeld's integral kernel (SIK) is obtained:

$$S_{00}^v(r_{2k}) \cong R_\infty K_0(r_{2k}) + (R_0 - R_\infty) \underline{\gamma}_0 L(r_{2k}) \,, \quad (13)$$

where $L(r_{2k})$ is the new integral kernel defined as:

$$L(r_{2kv}) = \int_{v=z+z_k'}^{\infty} K_0(r_{2kv}) \mathrm{d}v \,. \quad (14)$$

Cymomotive force

Once the UCD is determined, the radiation field ([1]) and the cymomotive force ([2], [3]), can be evaluated using the following definition expressions:

$$\vec{E}_{zr} = -\underline{\gamma}_0^2 [\hat{r} \times (\vec{\Pi}_0^{zr} \times \hat{r})] = -\underline{\gamma}_0^2 \Pi_{z0} \hat{e} \,, \quad (15)$$

$$CMF = |E_\theta r| \,, \quad (16)$$

where: $\hat{e} = \hat{r} \times (\hat{z} \times \hat{r}) = -\sin\theta \hat{\theta}$.

The Hertz's vector potential, $\vec{\Pi}_0^{zr} = \Pi_{z0} \hat{z}$, in the far field zone is in the following form:

$$\Pi_{z0} = \sum_{k=1}^{2} \int_{s_k'=0}^{l_k} \frac{I_k(s_k')}{4\pi \underline{\sigma}_0} [K_0(r_{1k}) + R_p(\underline{n}, \theta) K_0(r_{2k})] \mathrm{d}s_k' \,. \quad (17)$$

Current distribution along the antenna conductors is denoted by $I_k(s_k')$, whilst $K_0(r_{1k})$ and $K_0(r_{2k})$ are standard potential kernels from the original and the image in the flat mirror, respectively. $R_p(\underline{n}, \theta)$ is the reflection coefficient of the parallel polarized plane wave:

$$K_0(r_{1k}) = \frac{e^{-\underline{\gamma}_0 r_{1k}}}{r_{1k}} \,,\; r_{1k} = r - z_k' \cos\theta \,, \quad (18)$$

$$K_0(r_{2k}) = \frac{e^{-\underline{\gamma}_0 r_{2k}}}{r_{2k}} \,,\; r_{2k} = r + z_k' \cos\theta \,, \quad (19)$$

$$R_p(\underline{n}, \theta) = \frac{\underline{Z}_0 \cos\theta - \underline{Z}_1 \cos\theta_1}{\underline{Z}_0 \cos\theta + \underline{Z}_1 \cos\theta_1} \,, \quad (20)$$

where $\underline{Z}_0 = \sqrt{\mu_0 / \varepsilon_0}$ and $\underline{Z}_1 = \sqrt{\mu_0 / \underline{\varepsilon}_{r1} \varepsilon_0}$ are characteristic imedance of the air and ground, respectively.

Substituting (18-19) into (17), the following is obtained:

$$\Pi_{z0} = \frac{e^{-\underline{\gamma}_0 r}}{r} \sum_{k=1}^{2} \int_{s_k'=0}^{l_k} \frac{I_k(s_k')}{4\pi \underline{\sigma}_0} \left[e^{\underline{\gamma}_0 z_k' \cos\theta} + R_p(\underline{n}, \theta) e^{-\underline{\gamma}_0 z_k' \cos\theta} \right] \mathrm{d}s_k' \,. \quad (21)$$

Finally, adopting (21), and then substituting (15) into (16), the following expression is obtained for the CMF:

$$CMF = \left|E_\theta\, r\right| = \left|60 e^{-\gamma_0 r} F_{zr}(\theta)\right|, \qquad (22)$$

where $F_{zr}(\theta)$ is the field pattern defined as in [6].

NUMERICAL RESULTS

In order to illustrate the accuracy of the applied SIK approximation, results obtained for the modulus of the normalized SIK, $S_{00}^{v}(r_{2k})/\beta_0$, versus normalized radial distance, $Log(\beta_0\rho)$, are presented in Fig.2. The location of the vertical Hertz's dipole (VHD), z_k', is taken as a parameter. The results obtained by accurate calculations from [7] (solid down triangle), are also shown in the same figure for the sake of comparing. As it can be noted, the excellent accordance of the results is evident.

Second group of results addresses the cymomotive force determined for the case of the VDA placed above a lossy half-space of known electrical parameters. The considered VDA is of equal conductor length $l_1 = l_2 = 0.25\,\lambda_0$, and circular cross-section of radius $a_1 = a_2 = 0.007\,\lambda_0$, and feeding point at height $h = 0.25\,\lambda_0$.

The CMF versus spherical angle θ, for different values of the relative permittivity of the ground, is shown in Fig.3. Figs. 3a and 3b correspond to different values of the conductivity: $\sigma_1 = 10^{-3}$ S/m and $\sigma_1 = 10^{-2}$ S/m, respectively.

In Fig.4, the CMF versus spherical angle θ, for different values of the ground conductivity, is given. The results are presented in Figs. 4a, b and c, for three different values of the relative permittivity of the ground: $\varepsilon_{r1} = 1$, $\varepsilon_{r1} = 10$ and $\varepsilon_{r1} = 81$, respectively. The CMF calculated for the case of an ideally conducting ground is given in the same figures.

CONCLUSION

A simple, accurate and general model for approximation of the Sommerfeld's integral kernel was applied in this paper in order to determine the UCD of the vertical dipole antenna that is located above real ground, which is treated as a homogenous and isotropic lossy half-space.

Based on the theoretical analysis and presented numerical results, one can conclude that the applied SIK model has a simple form, that it is not limited by the values of electrical parameters of the ground, but yet, satisfyingly accurate in the surroundings of the VDA.

This implicates the possibility to completely characterize EM field structure of such wire structure or any other of similar kind (e.g. vertical mast antenna, vertical coupled antennas, etc.) in its near, as well as, far field zone. In this paper, the CMF, as one of the basic antenna characteristics, was analyzed in a wide range of electrical parameters of the ground.

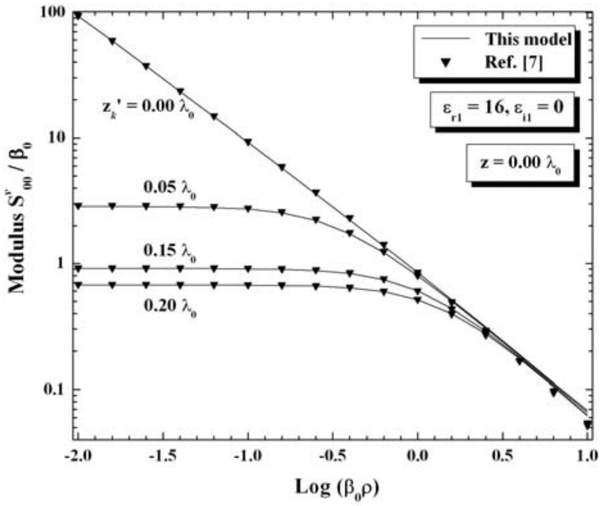

Fig.2 - *Modulus of the normalized SIK versus normalized radial distance. Relative permittivity is $\varepsilon_{r1} = 16$, and VHD position z_k' is parameter.*

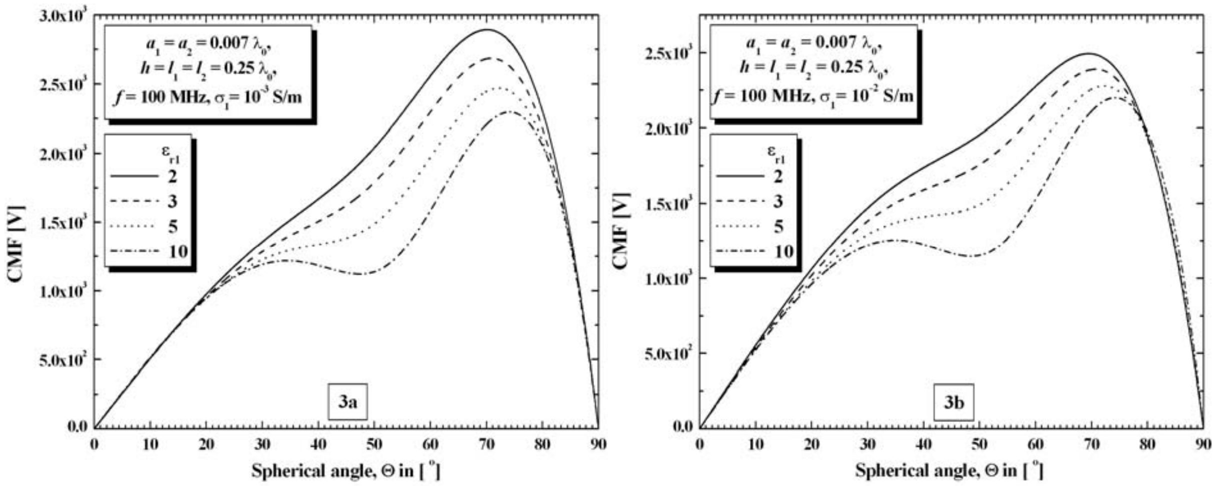

Fig.3 - *The CMF of the VDA versus spherical angle θ, for different values of the relative permittivity taken as parameter.*

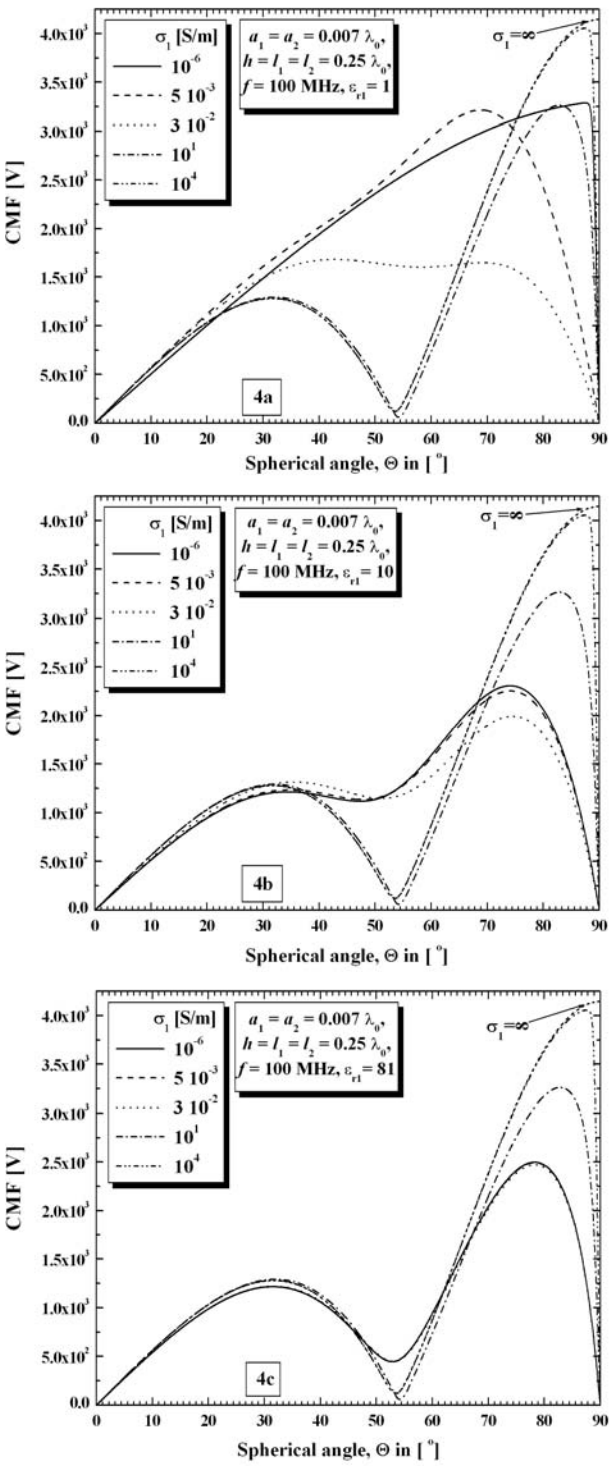

Fig.4 *- The CMF of the VDA versus spherical angle θ, for different values of the conductivity taken as parameter.*

Acknowledgement: *The author thanks prof. dr Dragutin Mitić for useful suggestions and comments during the preparation of this paper.*

REFERENCES

[1] M.B. Dragović: "Antennas and radio wave propagation", Electrotechnical Faculty of Belgrade, University of Belgrade, Belgrade 1996. (in Serbian)

[2] J.V. Surutka, D.N. Mitić: "Cymomotive force of MW emitting mast antenna", XIX ETAN, Vol.I, June 2-5, 1975, Ohrid, Yugoslavia, pp. 431-439. (in Serbian)

[3] V.L. Javor, D.N. Mitić: "Approximate evaluation of CMF of dipole antenna parallel to semi conducting medium", XXX ETRAN, Vol.V, June 2-6, 1986, Herceg Novi, Yugoslavia, pp.59-66. (in Serbian)

[4] D.M. Veličković: "A new polynomial approach to the numerical solution of thin cylindrical antenna problem", Archiv für Elektrotechnik, Vol. 5, 1973, pp. 223-236.

[5] M.P. Rančić, P.D. Rančić: "Vertical linear antennas in the presence of a lossy half-space: an improved approximate model", Int.J.Electron. Comun. AEÜ, Vol. 60, No. 5, 2006, pp. 376-386.

[6] M.P. Rančić, P.D. Rančić: "Field pattern of the vertical dipole antenna above a lossy half-space", 7th Int.Conf. on Applied Electromagnetics ΠEC 2005, 23-25 May, 2005, Niš, Serbia and Montenegro, pp. 121-122.

[7] V.V. Petrović: Private communications, ETF Belgrade, 2005.

Milica P. Rančić was born in Niš, Serbia, in 1977. She received the Dipl. ing. degree from the Faculty of Electronic Eng., University of Niš in 2002. From 2003. she has been engaged as a teaching assistant at the Dept. of Measurement, Faculty of Electrical Eng., University of Niš. Since September 2004., she is working at the Dept. of Theoretical Electrical Engineering at the same University, where she is enrolled to postgraduate studies. Her research area includes the wire structures' EM field modelling and photometry. M. Rančić is author or co-author of a number of published papers.