Regular paper

EQUIVALENT ELECTRODES METHOD APPLICATION ON ANISOTROPIC STRIPLINES CALCULATIONS

Nebojša B. RAIČEVIĆ [1], Saša S. ILIĆ [2]

Abstract: Theoretical analysis of a strip line, assuming quazi TEM mode propagation is carried out in the paper. The effective dielectric permittivity and characteristic impedance of transmission line is determined applying Equivalent Electrodes Method (EEM). The Green's function of the uniform charge per unit line length placed in an anisotropic media between two parallel plates is used. The cases of horizontal and vertical positions of the strip conductor are especially considered. Numerical results are given for the stripline with centred conductor, for the off-centered and for the broadside stripline.

Keywords: Equivalent Electrodes Method (EEM), anisotropic media, off-centered stripline, broadside stripline, characteristic impedance, Separating Variable Method (SVM).

1. INTRODUCTION

First, the uniform steady line charge q' placed parallel to two infinite, parallel plates having zero potential is considered (Fig. 1).

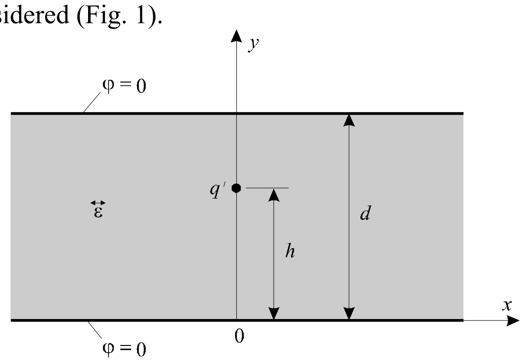

Fig. 1 – *Infinite line charge q' placed in an anisotropic medium, parallel to two infinite parallel plates.*

Distance between plates is $y = d$, and line charge q' is embedded on the height $y = h$.

Tensor of the dielectric permittivity has diagonal form:

$$\overleftrightarrow{\varepsilon} = \begin{bmatrix} \varepsilon_x & 0 \\ 0 & \varepsilon_y \end{bmatrix}. \tag{1}$$

The potential, φ, satisfies Poisson's equation:

$$\varepsilon_x \frac{\partial^2 \varphi}{\partial x^2} + \varepsilon_y \frac{\partial^2 \varphi}{\partial y^2} = -q'\delta(x)\delta(y-h), \tag{2}$$

including boundary conditions $\varphi = 0$ for $y = 0$ and $y = d$.

In this case it is possible to use SVM. The potential can be presented as one-dimensional functional trigonometric series

$$\varphi = \sum_{n=1}^{+\infty} X_n(x)\sin(k_n y), \tag{3}$$

where is

$$k_n = \frac{n\pi}{d}. \tag{4}$$

So, all boundary conditions on the walls are automatically satisfied. Functions $X_n(x)$ should be determined in the way that potential expression satisfies Poisson's equation.

Substituting (3) into (2), multiplying this expression with $\sin(k_m y)$ and integrating over y on segment $0 \le y \le d$, equation for $X_n(x)$ gets the form

$$X_n'' - (fk_n)^2 X_n = -\frac{2q'}{d\varepsilon_x}\sin(k_n h)\delta(x) \tag{5}$$

and solution of the previous equation is:

$$X_n(x) = \begin{cases} C_n\, e^{fk_n x} + D_n\, e^{-fk_n x}, & x < 0 \\ \left[C_n - \dfrac{q'\sin(k_n h)}{n\pi\varepsilon}\right] e^{fk_n x} + \left[D_n + \dfrac{q'\sin(k_n h)}{n\pi\varepsilon}\right] e^{-fk_n x}, & \text{for } x > 0 \end{cases} \tag{6}$$

where are $f = \sqrt{\dfrac{\varepsilon_y}{\varepsilon_x}}$ and $\varepsilon = \sqrt{\varepsilon_x \varepsilon_y}$.

When x coordinate tends to the infinity, electric potential is equal to zero. Taking into consideration this conditon it is possible to determine constants:

$$D_n = 0 \text{ and } C_n = \frac{q'\sin(k_n h)}{n\pi\varepsilon}. \tag{7}$$

So the expression for the potential is

$$\varphi = \varphi(x, y) = \frac{q'}{\pi\varepsilon}\sum_{n=1}^{+\infty} \frac{\sin(k_n h)\sin(k_n y)}{n} e^{-fk_n|x|}. \tag{8}$$

[1] University of Niš, Department of Electromagnetics, Aleksandra Medvedeva 14, 18000 Niš, Serbia, E-mail: nraicko@elfak.ni.ac.yu
[2] University of Niš, Department of Electromagnetics, Aleksandra Medvedeva 14, 18000 Niš, Serbia, E-mail: silic@elfak.ni.ac.yu

The electric field x component, E_x, can be expressed in the closed form:

$$E_x = \frac{q' f}{2d\varepsilon} \mathrm{e}^{-\frac{\pi}{d} fx} \left\{ \frac{\cos\left[\frac{\pi}{d}(y-h)\right] - \mathrm{e}^{-\frac{\pi}{d} fx}}{1 - 2\mathrm{e}^{-\frac{\pi}{d} fx} \cos\left[\frac{\pi}{d}(y-h)\right] + \mathrm{e}^{-\frac{2\pi}{d} fx}} - \frac{\cos\left[\frac{\pi}{d}(y+h)\right] - \mathrm{e}^{-\frac{\pi}{d} fx}}{1 - 2\mathrm{e}^{-\frac{\pi}{d} fx} \cos\left[\frac{\pi}{d}(y+h)\right] + \mathrm{e}^{-\frac{2\pi}{d} fx}} \right\}. \qquad (9)$$

Based on previous expression, the potential is finally:

$$\varphi = \frac{q'}{2\pi\varepsilon} \ln \sqrt{\frac{\mathrm{ch}\left(\frac{\pi}{d} fx\right) - \cos\left[\frac{\pi}{d}(y+h)\right]}{\mathrm{ch}\left(\frac{\pi}{d} fx\right) - \cos\left[\frac{\pi}{d}(y-h)\right]}} . \qquad (10)$$

2. THE EQUIVALENT ELECTRODES METHOD APPLICATION

Infinite linear electrodes are used as equivalent electrodes, having circular cross-section radius $a_\mathrm{e} = \frac{w}{4N}$, with central lines located at the places:

$$x_n = \pm (2n-1) \frac{w}{2N} \,;\, y_n = h \,, \qquad (11)$$

where $n = 1, 2, \ldots, N$.

3. NUMERICAL RESULTS

Effective dielectric permittivity and characteristic impedance of striplines with anisotropic media, for different line parameters, are presented in Tab. I, II, III, IV, V, VI and VII. Values obtained for isotropic materials are compared with corresponding results determined by using program package TXline.

Results are given for anisotropic materials:

A) Sapphire ($\varepsilon_{\mathrm{r}x} = 9.4\,; \varepsilon_{\mathrm{r}y} = 11.6$);

B) Boron-nitride [BN] ($\varepsilon_{\mathrm{r}x} = 5.12\,; \varepsilon_{\mathrm{r}y} = 3.4$);

C) Epsilam ($\varepsilon_{\mathrm{r}x} = 13\,; \varepsilon_{\mathrm{r}y} = 11.6$),

and for isotropic materials:

D) Alumina [Al_2O_3] ($\varepsilon_{\mathrm{r}x} = \varepsilon_{\mathrm{r}y} = 9.6$); and

E) Gallium-Arsenide [GaAs] ($\varepsilon_{\mathrm{r}x} = \varepsilon_{\mathrm{r}y} = 12.9$).

3. 1. Centered Stripline

Next the stripline with horizontally placed, strip conductor, having width w and thickness t is considered (Fig. 2).

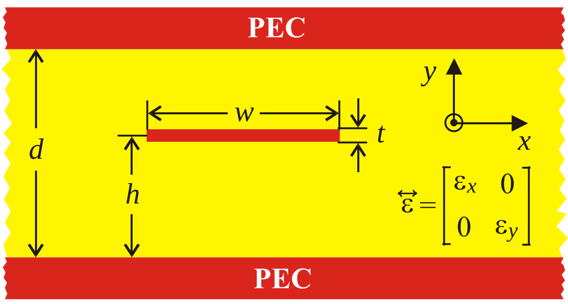

Fig.2 – *Off-centered stripline.*

Characteristic impedance of this stripline, Z_c, versus ratio t/w, for parameters $w/d = 0.75$ and $h = d/2$, is presented in Tab. I.

Table I

Characteristic impedance of centered stripline, Z_c $[\Omega]$ *versus ratio* t/w.

t/w	A	B	C	D	E
0.0001	23.575	41.110	22.925	25.475 (25.493)	21.981 (21.992)
0.001	23.561	40.901	22.870	25.419 (25.427)	21.928 (21.935)
0.01	23.209	40.373	22.522	25.021 (24.916)	21.591 (21.494)
0.1	20.698	35.850	20.069	22.289 (21.468)	19.228 (18.521)

Numerical results are given for anisotropic (columns A, B, C) and isotropic materials (columns D and E). In brackets are values calculated by TXline program package.

Relative value of effective dielectric permittivity, $\varepsilon_\mathrm{r}^\mathrm{eff}$, for this stripline, is shown in Tab. II.

Table II

Relative effective dielectric permittivity, $\varepsilon_\mathrm{r}^\mathrm{eff}$, *versus ratio* t/w.

t/w	A	B	C
0.0001	11.162	3.686	11.853
0.001	11.168	3.689	11.853
0.01	11.164	3.689	11.856
0.1	11.132	3.711	11.882

Table III

Characteristic impedance of centered stripline, Z_c $[\Omega]$, *versus ratio* w/d, *for* $t/w = 0.01$.

w/d	A	B	E	E (TXline)
0.25	41.857	70.493	38.499	37.591
0.50	29.711	51.002	27.513	27.211
0.75	23.209	40.373	21.591	21.494
1.00	19.049	33.451	17.777	17.779
1.50	13.999	24.882	13.115	13.217
2.00	11.040	19.767	10.368	10.523
4.00	5.904	10.707	5.567	5.815

In the extreme cases, for $\varepsilon_{\mathrm{r}x} = 2$ and $\varepsilon_{\mathrm{r}y} = 80$, for effective permittivity and characteristic impedance, the following values have been obtained: $\varepsilon_r^{eff} = 52.3093$ and $Z_c = 9.5490\,\Omega$, respectively. Next considered case is for $\varepsilon_{rx} = 80$ and $\varepsilon_{ry} = 2$ and then is $\varepsilon_r^{eff} = 6.0380$ and $Z_c = 28.1045\ \Omega$.

Influence of the strip conductor width on characteristic impedance is illustrated at Fig 3. Results are given for anisotropic materials A and B.

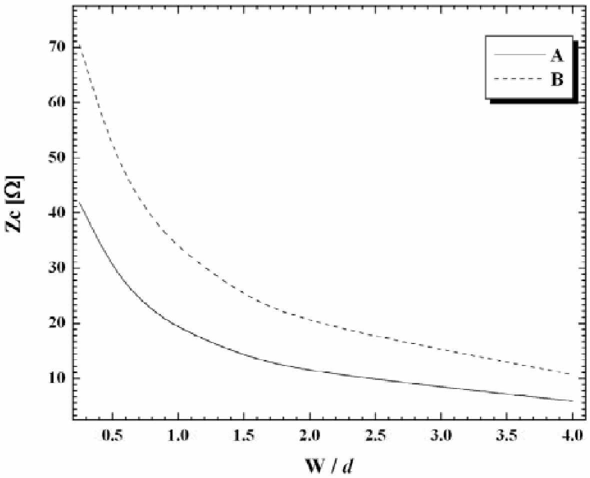

Fig.3 – *Characteristic impedance of centered stripline,* Z_c, *versus ratio* w/d.

3.2. Off -Centred Stripline

Stripline with horizontally, arbitrary placed strip conductor, having width $w/d = 1$ and thickness $t/w = 0.01$ is also analyzed.

Characteristic impedance as the function of strip conductor position, for anisotropic stripline, is presented in Tab. IV.

Table IV

Characteristic impedance of off-centered stripline, $Z_c[\Omega]$ *versus ratio* h/d.

h/d	A	B	C	$\varepsilon_{rx} = 2$ $\varepsilon_{ry} = 80$	$\varepsilon_{rx} = 80$ $\varepsilon_{ry} = 2$
0.05	4.162	7.541	4.122	1.747	8.057
0.10	7.838	14.071	7.728	3.288	13.839
0.15	10.734	19.214	10.588	4.567	17.975
0.20	13.140	23.323	12.883	5.628	21.092
0.25	15.037	26.592	14.715	6.497	23.479
0.30	16.525	29.146	16.151	7.191	25.295
0.35	17.647	31.064	17.229	7.718	26.635
0.40	18.432	32.401	17.983	8.091	27.557
0.45	18.896	33.191	18.428	8.313	28.098
0.50	19.049	33.451	18.576	8.386	28.276

Characteristic impedance changing is given for anisotropic materials A, B, C, and for special materials: $\varepsilon_{rx} = 2$ $\varepsilon_{ry} = 80$ (F); $\varepsilon_{rx} = 80$ $\varepsilon_{ry} = 2$ (G).

Those cases are shown on Fig. 4.

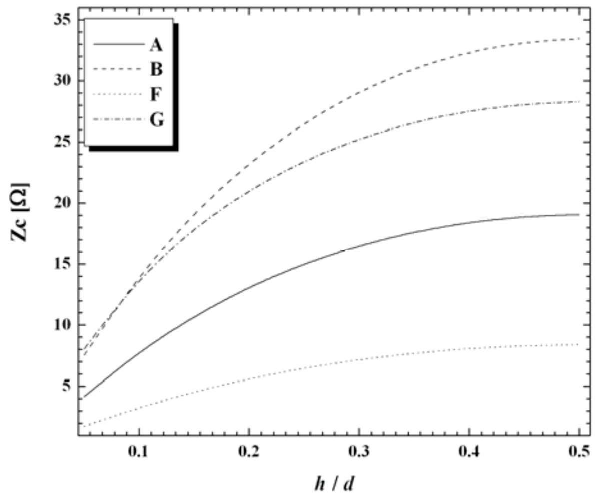

Fig.4 – *Characteristic impedance of off-centered stripline,* $Z_c[\Omega]$, *versus position of strip conductor,* h/d.

3.3. Broadside Stripline

Next considered problem is the broadside stripline calculation. Conductor is placed vertically, on the distance s from the bottom plate. Thickness of the strip conductor is t and the width is w.

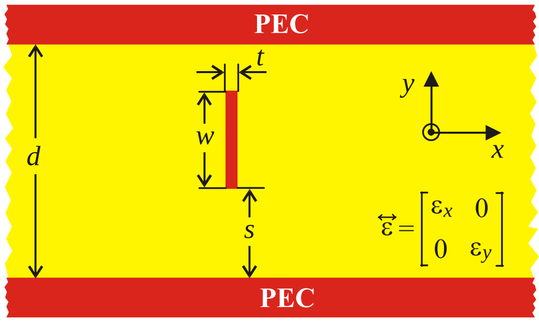

Fig.5 – *Broadside stripline.*

Influence of the conductor thickness, t/w, on characteristic impedance is shown in Tab. V, where is $w/d = 0.5$ and $s/d = 0.25$.

Table V

Characteristic impedance of broadside stripline, $Z_c[\Omega]$, *versus ratio* t/w.

t/w	A	B	E
0.0001	29.079	46.046	26.171
0.001	29.011	45.941	26.110
0.01	28.558	45.305	25.721
0.1	25.569	41.073	23.134

Changing of position of the strip conductor, s/d, influences on its characteristic impedance and this is illustrated in Tab. VI. In this case is $w/d = 0.3$ and $t/w = 0.01$. Characteristic impedance of the broadside stripline, $Z_c[\Omega]$, versus ratio w/d, for $s/d = 0.2$, is presented in Tab. VII.

Table VI

Characteristic impedance of the broadside stripline, $Z_c[\Omega]$, *versus ratio* s/d.

s/d	A	B	E
0.05	25.939	41.187	23.369
0.10	30.978	49.141	27.899
0.15	34.162	54.171	30.762
0.20	36.307	57.562	32.692
0.25	37.711	59.782	33.954
0.30	38.511	61.046	34.674
0.35	38.771	61.424	34.908

Table VII

Characteristic impedance of broadside stripline, $Z_c[\Omega]$, *versus ratio* w/d.

w/d	A	B	E
0.1	52.941	83.873	47.656
0.2	42.301	67.043	38.084
0.3	36.307	57.562	32.692
0.4	31.937	50.650	28.760
0.5	28.221	44.771	25.416
0.6	24.598	39.044	22.158
0.7	20.357	32.347	18.345

First two columns are illustrated by curves A and B (Fig.6).

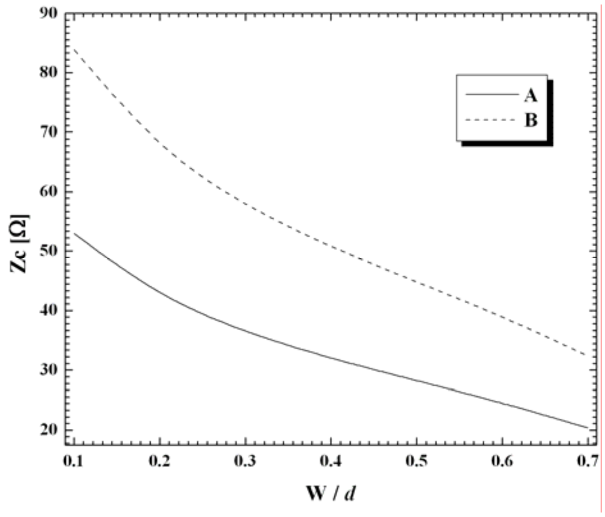

Fig.6 – *Characteristic impedance of broadside stripline,* $Z_c[\Omega]$, *versus ratio* w/d.

4. CONCLUSION

The aim of this paper is to research the influence of the strip conductor placed in anisotropic media between the two grounded perfect conducting (PEC) plates, on effective dielectric permittivity and characteristic impedance of the system into quazi TEM regime. For solving this problem, equivalent electrodes method (EEM) as very simple, powerful and accurate procedure is used.

It is necessary to know Green's function of uniform line charge placed in anisotropic dielectric media, between two infinite perfectly conducting grounded plates. This function is determined in the closed form. Afterwards, potential is approximately numerically determined by using equivalent electrodes method. EEM is applied in a standard way.

Also, quazi TEM theoretical analysis of the stripline is presented. The obtained numerical results for isotropic dielectric media are compared with corresponding results calculated by TXline program package and very good agreement can be noticed. It is significant to emphases that the EEM computer time calculation is litle longer than the computer time of the TXline program package calculation, because program TXline using analytical formulas for characteristic impedance and effective permittivity.

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Nebojša B. Raičević was born in Niš, Serbia, on March 27, 1965. He received the Diploma (with honors) and M. Sc. degrees at Faculty of Electrical Engineering from the University of Niš. He is currently a Research and Teaching Assistant at the Faculty of Electronic Engineering, Department of Electromagnetics, University of Nis.

His research interests include cable terminations and joints, numerical methods for electromagnetic problems solving, nonlinear electrostatic problems and magnetic field calculation of coils and permanent magnets.