Regular paper

CALCULATION OF CURRENT DENSITY DISTRIBUTION IN THE CONDUCTOR WITH RECTANGULAR CROSS-SECTION IN THE FERROMAGNETIC BLOCK GROOVE

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Abstract: This paper represents the results of calculating current suppression in the conductor with rectangular cross-section in the groove of ferromagnetic block. The calculation is carried out by applying simplified analytical formulas, loop method and FEMM programme package. When applying loop method, the effects of ferromagnetic block will be determined by means of mirror image theorem. Then the obtained results are compared.

Keywords: Loop method, Current density distribution, Ferromagnetic block groove.

INTRODUCTION

Current in the conductor with rectangular cross-section, which is situated in the ferromagnetic block groove, creates magnetic field in its own surrounding. Ferromagnetic material has considerable influence on the lines of magnetic field intensity. The lines of magnetic field intensity tend to close along the path of lower magnetic resistance, so their distribution is asymmetric regarding the horizontal symmetry axis of the conductor shown in Figure 1.

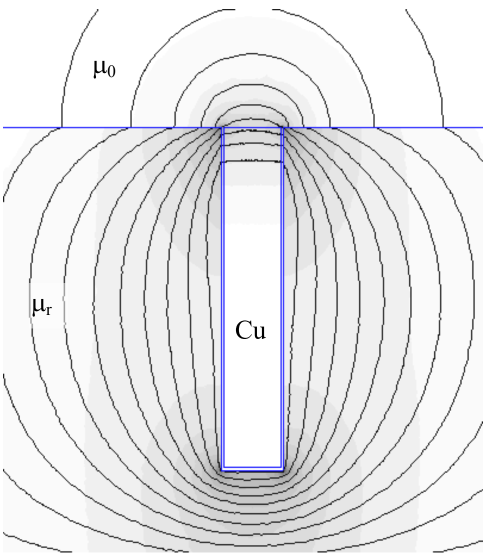

Fig. 1 – *Field lines in the surrounding of rectangular cross-section conductor in the groove of ferromagnetic block*

If alternate current is established in the conductor, it will lead to uneven distribution of electric filed caused by self-inductance along the conductor cross-section, which will, consequently, lead to uneven current distribution.

The current is distributed in the way that its density should rise when approaching the groove hole. This phenomenon leads to the increase of effective resistance of the conductor and to the increase of loss due to Joule's effect. If the conductor height has considerably higher values than its width, and since μ_{Fe} is higher than μ_0, the lines of magnetic field intensity in the groove are approximately perpendicular to the side face of the groove and they can be presumed to be parallel to axis y. Following the path of lower resistance, the lines close through ferromagnetic material by-passing the groove from the lower side, as it is shown in Figure 1.

According to Ampere’s Law, circulation of the vector $\boldsymbol{H}$ along the closed contour which is in concordance with the field line is approximately

$$\oint_l \boldsymbol{H}\mathrm{d}\boldsymbol{l} \approx H(x)b$$

Approaching the groove hole, the total current involved in circulation line increases, so the intensity of magnetic field increases.

If there is only one conductor in the groove, the intensity of magnetic field and current density are determined on the basis of the simplified analytical equations shown in [1].

$$\underline{H} = \frac{I}{b\,\mathrm{sh}\underline{\gamma}h}\mathrm{sh}\underline{\gamma}x$$

and

$$\underline{J} = \frac{I}{ah}\frac{\underline{\gamma}h}{\mathrm{sh}\underline{\gamma}h}\mathrm{ch}\underline{\gamma}x$$

The modules of magnetic field intensity and current density are determined on the basis of:

$$H = \frac{I}{b}\sqrt{\frac{\mathrm{ch}2kx - \cos 2kx}{\mathrm{ch}2kh - \cos 2kh}}$$

and

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$$J = \frac{I}{a}\sqrt{\omega\mu_0\sigma\frac{a}{b}}\sqrt{\frac{\text{ch}2kx + \cos 2kx}{\text{ch}2kh - \cos 2kh}}$$

where:

$\underline{I}$ - current in the conductor with rectangular cross-section,
a - conductor width,
b - groove width,
h - conductor height,

$$k = \sqrt{\frac{a}{b}\frac{\omega\mu_0\sigma}{2}}$$

and

$$\underline{\gamma} = (1 + j)k \, .$$

The last two equations represent approximate formulas for calculating the module of magnetic field intensity and module of current density along the cross-section of rectangular conductor in ferromagnetic block groove.

LOOP METHOD

Loop method [2] means to replace square cross-section conductor with the series of straight, parallel circular cross-section conductors. These conductors form current loops. The electrodynamic equation should be written for each loop taking into account all the currents in the series of conductors. The needed number of loops depends on the form of conductor cross-section, that is to say, on the degree of its symmetry. The currents of certain conductors in the series can be obtained by calculating the formed system of equations.

The general form of loop equations is:

$$\underline{I}_a \underline{Z}_a - \underline{I}_b \underline{Z}_b + j\omega\underline{\Phi}_{ab} = 0$$

Where $\underline{I}_a$ and $\underline{I}_b$ are the currents in the conductors a and b, which form the loop $a - b$. $\underline{Z}_a$ and $\underline{Z}_b$ are the impedances of the conductors a and b, and $j\omega\underline{\Phi}_{ab}$ is inductive voltage in the loop $a - b$ caused by all the currents in the series of conductors.

When writing the equations of electrodynamic balance for each loop, it is considered that all the currents have the same direction. The direction in the loop is determined by the direction of the current in the first conductor of the loop. In the same way the sign of magnetic flux through the loop is determined.

When determining self-inductances, the effect of closeness was also taken into consideration.

The distribution of current density along the cross-section of rectangular conductor can be determined in this way.

If the rectangular conductor is situated in groove of ferromagnetic block, its influence can be determined by applying mirror image theorem.

MIRROR IMAGE THEOREM

Straight, very long conductor with the current $\underline{I}$ in Fig.2, is situated in the air in the ferromagnetic surrounding with the relative permeability , and is parallel to dividing surface.

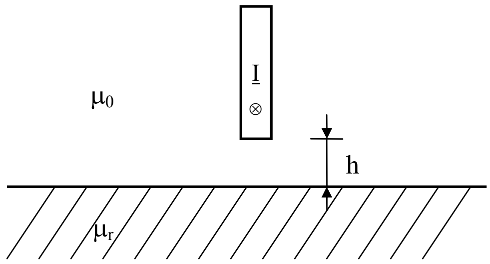

Fig. 2 - *Conductor with the current $\underline{I}$ in the ferromagnetic surrounding*

In order to calculate electromagnetic values in the area above ferromagnetic block, the influence of ferromagnetic surroundings can be replaced with the conductor with the current $\alpha\underline{I}$; the conductor, relating to the dividing surface, is set on the place of mirror image Fig.3.

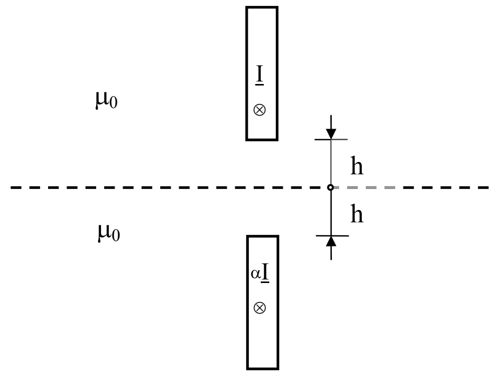

Fig.3 - *Conductor with the current $\underline{I}$ and its image*

Coefficient α is determined [3] as

$$\alpha = \frac{\mu_r - 1}{\mu_r + 1}.$$

Straight, rectangular conductor with the current $\underline{I}$ is situated in the deep groove of ferromagnetic block Fig. 4.

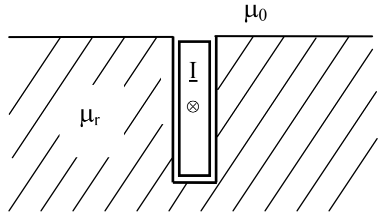

Fig.4 - *Rectangular conductor in the ferromagnetic block groove*

According to the mirror image theorem, the influence of ferromagnetic block can be, in the first approximation, replaced with five rectangular conductors with the currents $\alpha \underline{I}$, Fig.5 in order to determine the distribution of current density in the conductor.

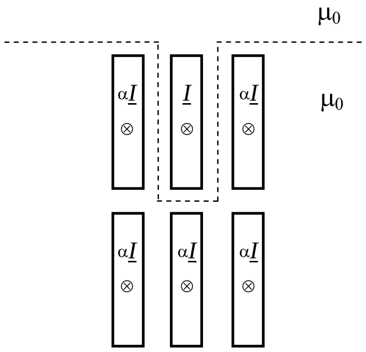

Fig.5 - *The influence of ferromagnetic surrounding replaced with the equivalent currents in the homogenous surrounding*

If we presume that the isolation thickness is negligibly low in relation to the dimensions of conductor cross-section, that is to say, the conductor rests on the groove sides and it is not in galvanic connection, and that relative magnetic permeability is $\mu_r \rangle\rangle 1$, so $\alpha \approx 1$, the calculation has been carried out according to the previously mentioned method.

CALCULATION RESULTS

We examined the rectangular cross-section conductor, with the dimensions $(10 \times 60)\text{mm}^2$ in which the current is $\underline{I} = (600 + j0)\text{A}$. Relative magnetic permeability of ferromagnetic block is $\mu_r = 10^6$.

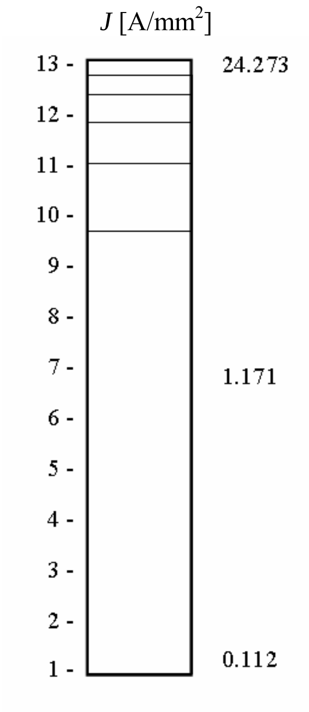

Fig. 6 - *Distribution of current density obtained by applying approximate calculation equation.*

The groove width in ferromagnetic block is slightly higher than the conductor width and it is $b = 10.2\,mm$.

On the basis of calculation by means of approximate analytical equations, current distribution in copper conductor of rectangular cross-section shown in Figure 6 is as follows:

According to loop method, current density distribution is shown in Figure 7.

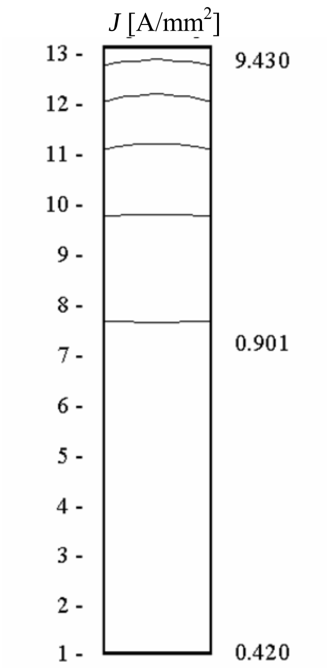

Fig. 7 *Distribution of current density obtained by means of loop method*

Distribution of current density shown in Figure 8 is obtained by using FEMM programme package.

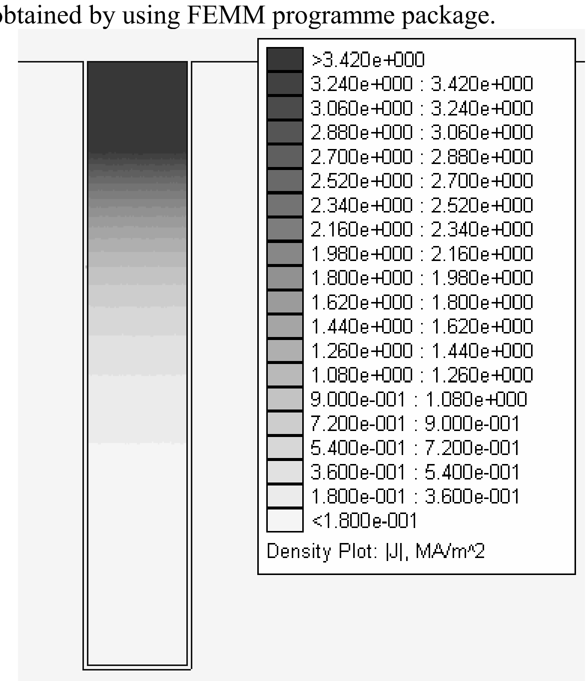

Fig. 8 - *Distribution of current density obtained by applying FEMM programme package*

RESULTS COMPARISON

The results shown in Table I are obtained on the basis of calculations carried out by applying three methods previously mentioned.

The results in the first column are obtained by means of approximate analytical equations; the results in the second column are obtained by using loop method; finally, the results in the third column are obtained by applying FEMM programme package.

The fourth column represents deviation of the first column results comparing to the third column results, while the fifth column shows the deviation of the second column results comparing to the third column ones.

Table I

	R_1 - Analytically method	R_2 - Loop method	R_3 - FEMM	100 $(R_1 - R_3)/R_3$	100 $(R_2 - R_3)/R_3$
1	0.112	0.42	0.32	-65.00%	31.25%
2	0.119	0.44	0.36	-66.94%	22.22%
3	0.147	0.48	0.45	-67.33%	6.67%
4	0.244	0.56	0.55	-55.64%	1.82%
5	0.42	0.69	0.67	-37.31%	2.99%
6	0.706	0.78	0.79	-10.63%	-1.27%
7	1.171	0.9	0.95	23.26%	-5.26%
8	1.938	2.05	2.105	-7.93%	-2.61%
9	3.211	3.35	3.21	0.03%	4.36%
10	5.324	4.5	4.35	22.39%	3.45%
11	8.828	5.95	5.7	54.88%	4.39%
12	14.639	7.5	7.1	106.18%	5.63%
13	24.273	9.43	8.56	183.56%	10.16%

Figure 9 represents graphic illustration of the results of calculating current density along the conductor vertical axis considering all three presented calculation methods.

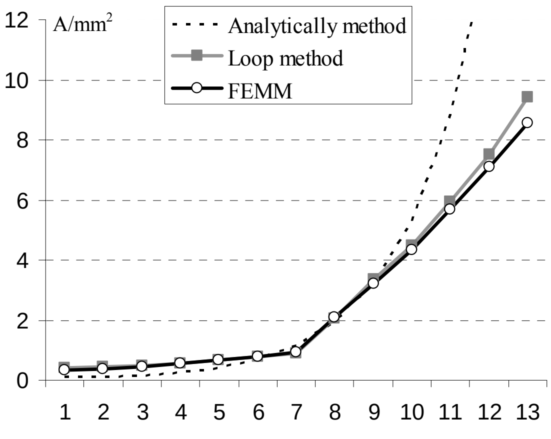

Fig. 9 - *Graph illustrating the results of calculating current density along conductor vertical axis*

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Milan Plazinić was born in Čačak, Serbia, in 1963. He graduated from Technical Faculty in Čačak, department of Electrical engineering, in 1987. He completed her postgraduate studies at Electronic Faculty in Nis at Department of Theoretical Electrical engineering and in 1998 he defended his master thesis. Now he is working his doctoral thesis from the theory of electromagnetic fields. His main research interests are calculations the values of the skin effect in the rectangular cross section conductors.