Regular paper

RING CROSS-SECTION SHAPE INFLUENCE ON “SATURN” CAPACITOR CAPACITANCE

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Abstract: In this paper, the capacitance calculation of “Saturn” capacitor is presented. The capacitance, when the capacitor ring has finite thickness and the ring cross-section has different shapes is calculated. The equivalent electrodes method and point-matching method are used for calculation. The obtained results are shown graphically and in the tables.

Keywords: Equivalent electrodes method (EEM), Point-matching method, Image theorem.

INTRODUCTION

In references [4, 5, 6] a capacitance of “Saturn” capacitor is considered. In papers [4, 5] a thin ring with negligible thickness has been observed. In the paper [6] the ring has finite thickness. The ring shape cross-section is rectangular.

An influence of ring cross-section shape is considered in this paper. The obtained results have been compared. Short discussion is given.

Equivalent electrodes method is used for determination an unknown capacitance. This method has been developed at the Faculty of Electronic Engineering in Niš, by Prof. Dragutin M. Veličković [1]. A basic idea of this method is replacing an electrostatic system by a finite system of equivalent electrodes (EE).

The equivalent electrodes of different shapes can be used depending on the problem geometry. The flat or oval strips (for plan-parallel problems), spherical bodies (for three-dimensional problems) or toroidal electrodes (for systems with axial symmetry) can be commonly used. The equivalent electrodes potential is equal to the real electrode potential. The system of linear equations is formed using this boundary condition, with equivalent electrodes charges as unknown values. After solving this system, the unknown charges of EE can be determined. Using standard formulas the potential, the electric field strength and the capacitance of the system can be computed.

OUTLINE OF THE METHOD

A sphere of radius a and ring of thickness δ, having inner radius b and exterior radius c, form “Saturn” capacitor [4, 6]. The ring is at the potential U and the sphere is at the potential V. A normal distance between the sphere centre and the ring is h (Fig. 1).

For determination the capacitance of this system, the ring is divided in N_j strips [3], where $j = 1, 2$, Fig. 2. The radius of n-th strip is

$$r_{jn} = 0.5b\left[\left(\frac{c}{b}\right)^{n/N_j} + \left(\frac{c}{b}\right)^{(n-1)/N_j}\right], \qquad (1)$$

and its width is

$$\Delta l_{jn} = b\left[\left(\frac{c}{b}\right)^{n/N_j} - \left(\frac{c}{b}\right)^{(n-1)/N_j}\right], \qquad (2)$$

where $j = 1, 2$.

Indexes 1 and 2 correspond to the strips on the top and upper ringside, respectively.

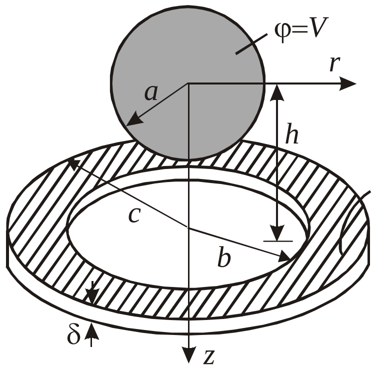

Fig. 1. – *”Saturn” capacitor*

A ring strips separation has been done in that way that ratio of strip radius and strip width for each strip is equal.

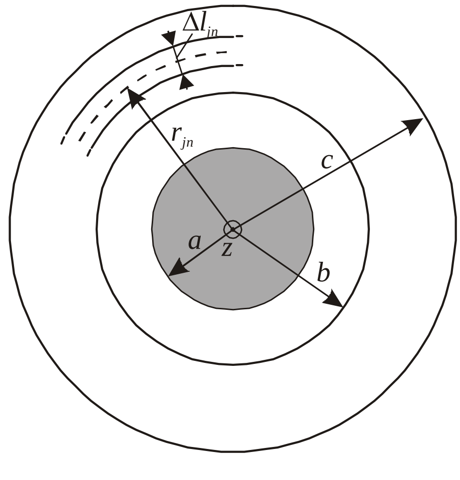

Fig. 2. – *EEM application*

If the ring cross-section shape is rectangular, inner and exterior sides with radius $r = b$ and $r = c$ have been divided in $N_3 = N_4$ ring strips, Fig. 3, with width

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$$\Delta l_{kn} = \frac{\delta}{N_k} \ , \qquad (3)$$

($n = 1, ..., N_3$), placed in positions

$$h_{kn} = h - \frac{\delta}{2} + (2n-1)\frac{\Delta l_{kn}}{2} \ . \qquad (4)$$

where $k = 3, 4$. Indexes 3 and 4 correspond to inner ($r_{3n} = b$) and exterior ($r_{4n} = c$) ringside, respectively.

The values of N_3 and N_4 are determined from expression

$$N_3 = N_4 = N_1 \frac{\delta}{c-b} \ . \qquad (5)$$

In that way, the ratio of the sides is proportional to the number of equivalent electrodes.

Each of the formed strips can be replaced by equivalent loops, having radius r_{kn} with circular cross-section of radius $a_{\mathrm{e}kn} = \Delta l_{kn} / 4$, ($k = 1, 2, 3, 4$).

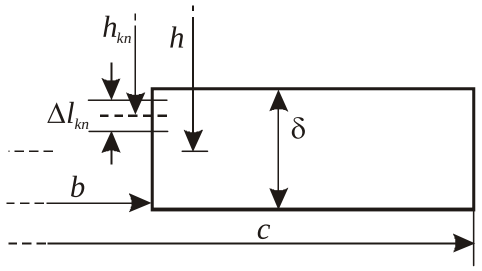


Fig. 3. – *Rectangular shape of the ring cross-section*

If the inner and exterior sides of ring cross-section are half circular, with radius $\delta / 2$, Fig. 4, they will be divided in $N_3 = N_4$ strips with arc $\delta\alpha$, where

$$2\alpha = \frac{\pi}{N_k} \ , \qquad (6)$$

where $k = 3, 4$. Indexes 3 and 4 correspond to inner and exterior sides of ring cross-section, respectively.

Each of the formed strips can be replaced by equivalent loops with circular cross-section of radius

$$a_{\mathrm{e}kn} = \frac{\delta}{2}\sin\left(\frac{\pi}{4N_k}\right) .$$

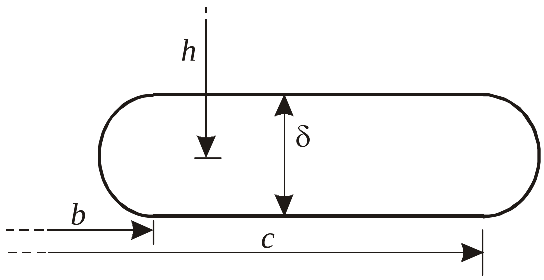


Fig. 4. – *Shape of the ring cross-section*

The radii of formed strips are

$$r_{kn} = b - \frac{\delta}{2}\cos\theta_{kn} \ \text{ and } \qquad (7)$$

$$r_{kn} = c + \frac{\delta}{2}\cos\theta_{kn} \ , \qquad (8)$$

for inner and exterior sides of ring cross-section, respectively. The normal distance of these strips from the sphere centre is

$$h_{kn} = h - \frac{\delta}{2}\sin\theta_{kn} \ , \qquad (9)$$

where

$$\theta_{kn} = \frac{\pi}{2N_k}(2n - 1 - N_k) \ , \qquad (10)$$

($n = 1, 2, ..., N_k$ and $k = 3, 4$).

Application of previous expressions depends on the ring cross-section shape. If the exterior side of ring cross-section is flat, the strips will be formed using the expression (4). If the inner side is half circular, the strips will have positions described using the expressions (7) and (9).

For any ring cross-section shape, the sphere and N loops, where $N = N_1 + N_2 + N_3 + N_4$, form the system.

Applying the image theorem in the sphere mirror, the equivalent system is formed. The charges of the loops, their images in the sphere mirror and one point charge placed in the centre of the sphere, Fig. 5, form this system.

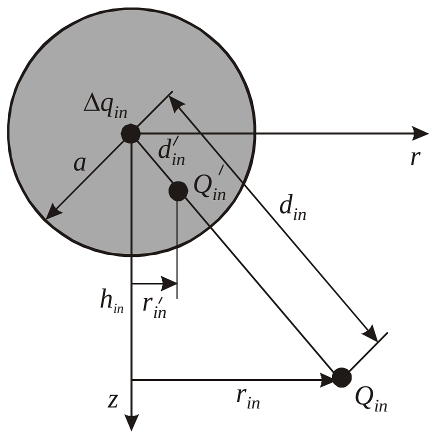


Fig. 5. – *Equivalent system*

The images are loops, too, with radii

$$r'_{in} = \frac{a^2}{r_{in}} \ , \qquad (11)$$

and

$$Q'_{in} = -\frac{a}{r_{in}} Q_{in} \qquad (12)$$

is charge of the n -th loop image ($i = 1, 2, 3, 4$).

The ring and the sphere form a capacitor, so their charges must be equal, but of the opposite sign. Because of that, the total sphere charge must be $-Q$ [3]. One part of this charge is divided into image charges, obtained using image theorem.

The influence of these charges can be presented with one equivalent loop, and other charges

$$\Delta q_{in} = -Q_{in} - Q'_{in} = -\left(1 - \frac{a}{r_{in}}\right) Q_{in}, \qquad (13)$$

($i = 1, 2, 3, 4$), are uniformly placed at the sphere surface, so their influence can be presented with point charge placed in the centre of the sphere. Its intensity is

$$\Delta q_i = \sum_{n=1}^{N_i} \Delta q_{in} . \qquad (14)$$

Parameter N_i ($i = 1, 2, 3, 4$) corresponds to the equivalent electrodes number of each ringside.

The potential at point $\mathrm{M}(r, z)$ is

$$\varphi = \sum_{i=1}^{4} \sum_{n=1}^{N_i} \frac{1}{2\pi^2 \varepsilon} \left[Q_{in} \frac{\mathrm{K}\left(\frac{\pi}{2}, k_{in}\right)}{\sqrt{(r + r_{in})^2 + z^2}} + \right.$$

$$\left. + Q'_{in} \frac{\mathrm{K}\left(\frac{\pi}{2}, k'_{in}\right)}{\sqrt{(r + r'_{in})^2 + z^2}} + \frac{\pi}{2} \frac{\Delta q_{in}}{\sqrt{r^2 + z^2}} \right], \qquad (15)$$

where $\mathrm{K}\left(\frac{\pi}{2}, k_{in}\right)$ and $\mathrm{K}\left(\frac{\pi}{2}, k'_{in}\right)$ are complete elliptic integrals of the first kind, with modulus

$$k_{in}^2 = \frac{4 r r_{in}}{(r + r_{in})^2 + z^2} \text{ and } k'^2_{in} = \frac{4 r r'_{in}}{(r + r'_{in})^2 + z^2} .$$

The unknown charges Q_{in} can be determined when the potential (15) is matched in N matching points placed at the electrodes surfaces.

After solving formed system of linear equations the capacitance can be calculated as

$$C = \frac{Q}{U - V} , \qquad (16)$$

where

$$Q = \sum_{n=1}^{N} Q_n \qquad (17)$$

is the total charge of the ring electrode.

The sphere potential, denoted with V, derives from the charge Δq_{in} placed in the centre of the sphere. Using condition that the sphere is equipotential this potential can be determined.

NUMERICAL RESULTS

In Table I and Table II, the capacitance values, for different number of equivalent electrodes and different values of parameters, are shown. The ring shape is rectangular as in Fig.3.

The ring thickness is given using the parameter Δ, where

$$\Delta = \frac{\delta}{c - b} . \qquad (18)$$

From these tables the good convergence of the results can be noticed.

In Tables III-VII, the capacitance values for different shapes of ring cross-section and different values of parameters are shown. The rectangular cross-section is “Shape 1”. “Shape 3” corresponds to the shape presented in Fig.4. For “Shape 2” the inner side is half circular and exterior side of ring cross-section is flat.

Table I

Capacitance values for different number of equivalent electrodes, for $b/a = 2.0$, $c/a = 3.0$ and $h/a = 0.0$.

$\Delta = 0.01$		$\Delta = 0.1$	
$N_1 = N_2$	$C/2\pi^2\varepsilon a$	$N_1 = N_2$	$C/2\pi^2\varepsilon a$
1	0.6619838292	1	0.7809054320
5	0.8317598138	5	0.8898828318
10	0.8501499884	10	0.8891853916
30	0.8592014113	30	0.8883947001
50	0.8601887498	50	0.8880555296
80	0.8614163816	80	0.8878126893
100	0.8613458586	100	0.8877205062

Table II

Capacitance values for different number of equivalent electrodes, for $b/a = 2.0$, $c/a = 3.0$ and $h/a = 1.0$.

$\Delta = 0.01$		$\Delta = 0.1$	
$N_1 = N_2$	$C/2\pi^2\varepsilon a$	$N_1 = N_2$	$C/2\pi^2\varepsilon a$
1	0.6259628454	1	0.7312439697
5	0.7690208297	5	0.8158960191
10	0.7838603850	10	0.8150518383
30	0.7912515676	30	0.8142989902
50	0.7919124213	50	0.8140160913
80	0.7928441785	80	0.8138183013
100	0.7927823051	100	0.8137440020

Table III

Capacitance values for different ring cross-section shapes, for $b/a = 2.0$, $c/a = 3.0$ $h/a = 0.0$ and $N_1 = N_2 = 80$.

	$C/2\pi^2\varepsilon a$		
Δ	*Shape 1*	*Shape 2*	*Shape 3*
0.01	0.86141638	0.86298929	0.86345126
0.05	0.87449230	0.88038378	0.88186750
0.10	0.88781269	0.89947574	0.90196902
0.15	0.89955271	0.91748543	0.92077521
0.20	0.91026868	0.93499720	0.93891125

Table IV

Capacitance values for different ring cross-section shapes, for $b/a = 2.0$, $c/a = 3.0$ $h/a = 0.5$ *and* $N_1 = N_2 = 80$.

	$C/2\pi^2\varepsilon a$		
Δ	*Shape 1*	*Shape 2*	*Shape 3*
0.01	0.84128942	0.84267839	0.84314911
0.05	0.85350081	0.85866952	0.86018905
0.10	0.86597669	0.87613654	0.87870532
0.15	0.87700153	0.89252608	0.89593571
0.20	0.88709392	0.90838106	0.91246228

Table V

Capacitance values for different ring cross-section shapes, for $b/a = 2.0$, $c/a = 3.0$ $h/a = 1.0$ *and* $N_1 = N_2 = 80$.

	$C/2\pi^2\varepsilon a$		
Δ	*Shape 1*	*Shape 2*	*Shape 3*
0.01	0.79284418	0.79385469	0.79434681
0.05	0.80319079	0.80688901	0.80849713
0.10	0.81381830	0.82095686	0.82371433
0.15	0.82325237	0.83398698	0.83769847
0.20	0.83193097	0.84643662	0.85094155

Table VI

Capacitance values for different ring cross-section shapes, for $b/a = 2.0$, $c/a = 3.0$ $h/a = 2.0$ *and* $N_1 = N_2 = 80$.

	$C/2\pi^2\varepsilon a$		
Δ	*Shape 1*	*Shape 2*	*Shape 3*
0.01	0.68669020	0.68715199	0.68768205
0.05	0.69392877	0.69554955	0.69732586
0.10	0.70137803	0.70436652	0.70750135
0.15	0.70799316	0.71230593	0.71664284
0.20	0.71408297	0.71969252	0.72509961

Table VII

Capacitance values for different ring cross-section shapes, for $b/a = 2.0$, $c/a = 3.0$ $h/a = 5.0$ *and* $N_1 = N_2 = 80$.

	$C/2\pi^2\varepsilon a$		
Δ	*Shape 1*	*Shape 2*	*Shape 3*
0.01	0.53392732	0.53407227	0.53455804
0.05	0.53832625	0.53880468	0.54047503
0.10	0.54278345	0.54360770	0.54664275
0.15	0.54668464	0.54780124	0.55211553
0.20	0.55022760	0.55159450	0.55711441

From these tables it can be found that if the distance between the sphere and the ring increases, the difference between capacitance results for different shapes of ring cross-section increases, too. Also, the bigger ring thickness, the bigger capacitance value is obtained. The smallest capacitance values are for the "Shape 1". When both sides of the ring cross-section are of half circular shape ("Shape 2"), the capacitance values are the biggest.

From Table VII it is evident that an influence of ring thickness is negligible. When the distance between ring and sphere is large, the sphere doesn't "see" the ring thickness. The distance between equivalent electrodes placed on the ringsides and their images in the sphere is approximately equal. That is the reason why ring thickness hasn't influence on the system capacitance.

In Figs. 6-8 the capacitance values for different parameters values are shown. All presented results are obtained when ring cross-section have the "Shape 1".

In Fig. 6 the capacitance dependence versus ring thickness, i.e. parameter Δ, when b/a and c/a have constant values, is shown.

From Fig. 6b it is evident that when the distance between the ring and the sphere is very big and the ring is of big thickness, the capacitance has constant value. As it is mentioned, in that case, the sphere doesn't "see" the ring thickness, so the capacitance is constant.

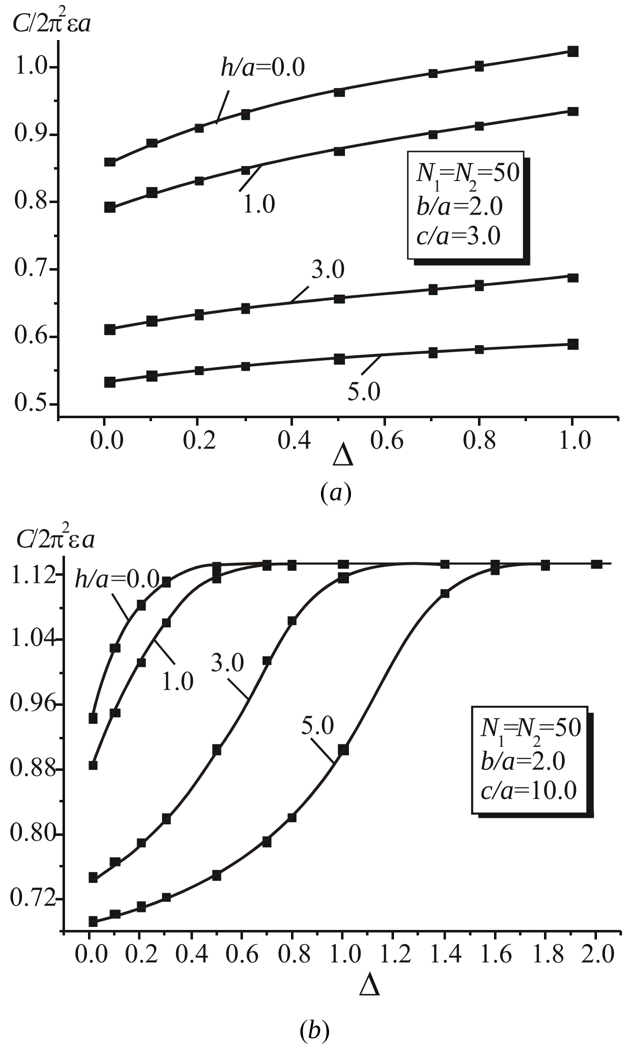
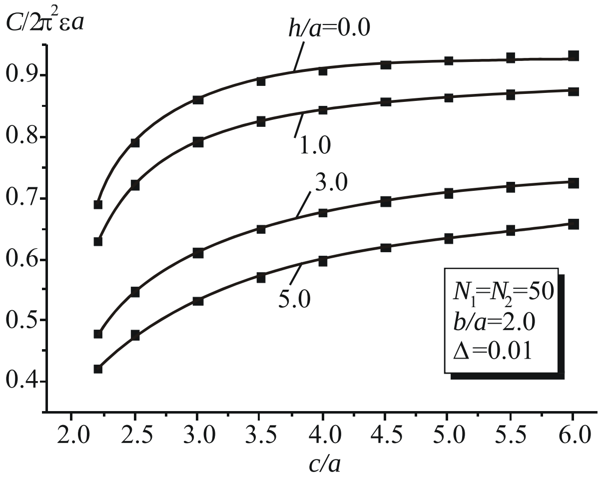


Fig. 6. – *Capacitance dependence versus parameter* Δ *for different values of parameter* h/a

In Fig. 7 the capacitance dependence versus parameter c/a, when b/a and Δ have constant values, is shown.



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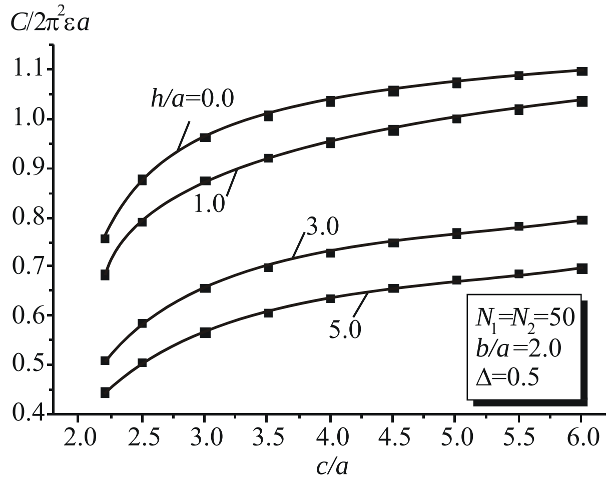
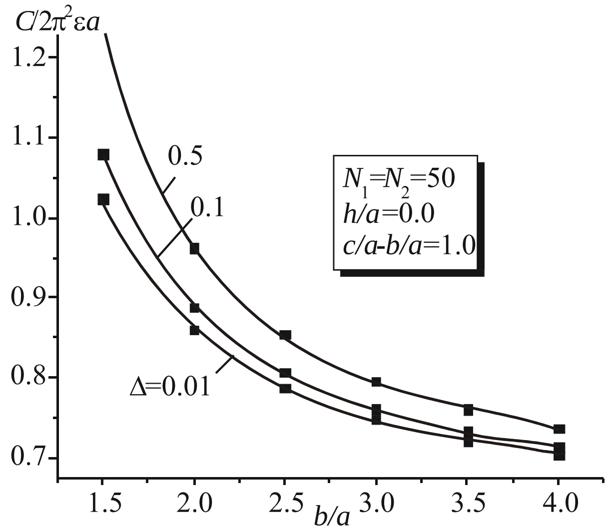


(*b*)

Fig. 7. – *Capacitance dependence versus parameter* c/a *for different values of parameter* h/a

In Fig. 8, parameters $c/a - b/a$ and h/a have constant values, and parameters b/a and Δ have different values. When the ring is so far from the sphere, for any value on the ring thickness, the capacitance stream to equal value.



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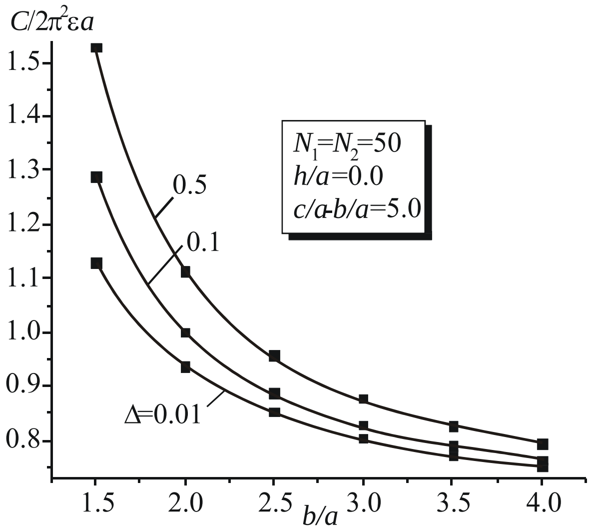

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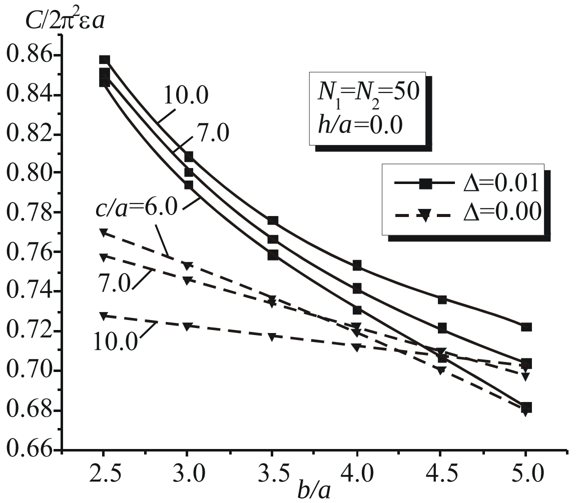
Fig. 8. – *Capacitance dependence versus parameter* b/a *for different values of parameter* Δ

The capacitance values for thin ring with negligible thickness and the ring with finite thickness have been compared in Fig. 9.



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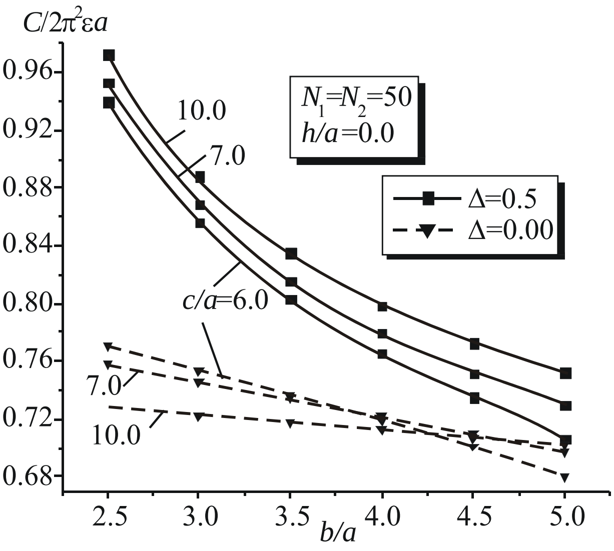

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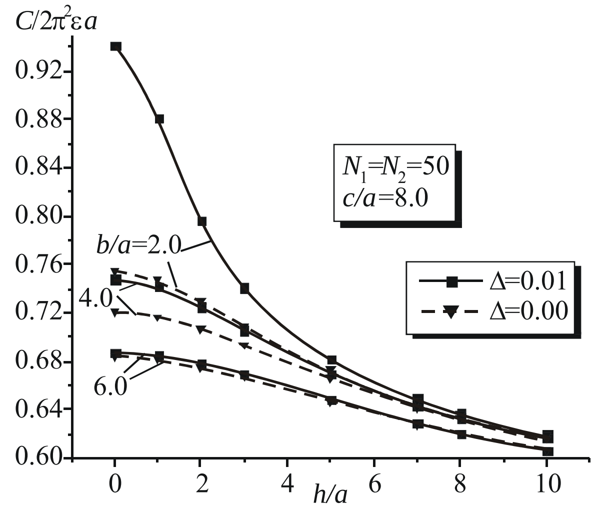

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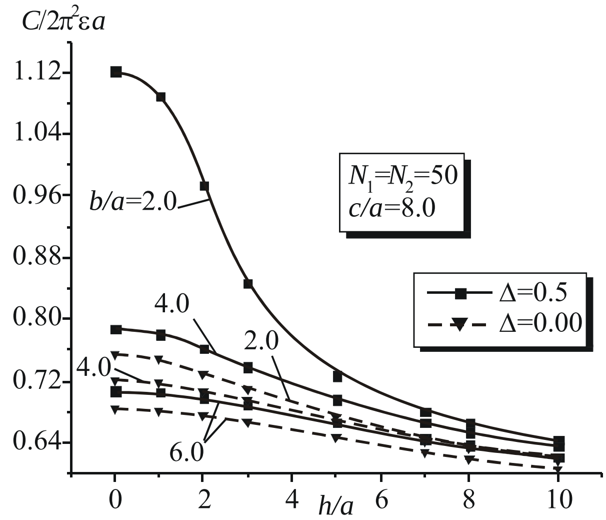

(*d*)

Fig. 9. – *Results comparison for different ring thickness and different values of parameters*

In Figs. 9c and 9d, it can be seen that for high distances between ring and sphere, capacitance values stream to equal value.

CONCLUSION

An application of the Equivalent electrodes method for calculation of the "Saturn" capacitor capacitance is presented in this paper. "Saturn" capacitor with different ring cross-section shapes is considered. The convergence of the results is shown in the tables, and the capacitance values for different values of parameters are shown graphically. Also, obtained results for capacitance for different ring cross-section shapes are shown in the tables.

The obtained results have shown that when the number of equivalent electrodes is small, the good convergence of the results is achieved. When the distance between ring and sphere is large, for any ring cross-section shape, the capacitance has approximately constant value.

The ring is divided in the strips using the expressions (1) and (2) because in the earlier investigation [2] is shown that the obtained error is smaller.

A theoretical approach for the capacitance calculation of "Saturn" capacitor is presented in this paper. The same procedure can be applied in the grounding theory.

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