Regular paper

ELECTROSTATIC FIELD ANALYSIS IN DIELECTRIC BODY WITH CAVITY

Mirjana T. PERIĆ[1], Bojana Z. NIKOLIĆ[2] and Zlata Ž. CVETKOVIĆ[3]

Abstract: A calculation of electrostatic field strength in a cylindrical dielectric body with arbitrary positioned and dimensioned cavity is presented in this paper. An influence of several parameters to the electrostatic field strength in the cavity is considered. The cross-section of dielectric body has a circular shape. The whole system is in the homogeneous transverse electrostatic field. These results, obtained by Charge simulation method, will be compared to the results obtained by most commonly used software in Electromagnetic.

Keywords: Electrostatic field, Charge simulation method, Boundary conditions, Penetrated electrostatic field.

INTRODUCTION

One of the mostly used methods for numerical electrostatic problems solving is Charge simulation method (CSM). Basic idea of the method is replacing the existing electrodes by fictitious charges (FCs), chosen in certain order and placed inside the electrodes volumes. The unknown intensities of FCs are determined to satisfy boundary conditions on the electrodes surfaces. In that way, system of linear equations with FCs as unknown values is formed. After solving this system, the unknown FCs can be determined. Using standard electrostatic formulas the potential and the electric field strength can be calculated. The correct choice of the type and the form of FCs is very important, especially with the respect to the realized accuracy and convergence with the number of FCs.

The problem of potential determination in dielectric bodies with cavities is considered also before at previous conferences [1, 3], international seminar [4], in magazine [2] and thesis [5]. Part of the results shown in this paper had been obtained at Technical University of Ilmenau, Germany [6].

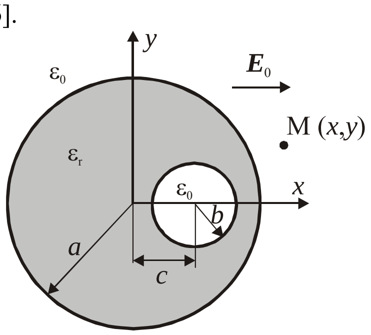

Fig. 1 - *Cross-section of cylindrical dielectric body with circular cavity*

In this paper the calculation of electrostatic field strength in a cylindrical dielectric body with eccentric circular cavity is considered, Fig. 1. The dielectric body has a circular cross-section. The whole system is placed in the homogeneous transverse electrostatic field, $\boldsymbol{E}_0 = E_0\hat{\boldsymbol{x}}$.

APPLICATION OF THE METHOD

In this particular case, the system is plan-parallel, so the line charges with constant density per unit length are used as FCs. The whole system is divided in three FCs systems.

The first FCs system is presented in Fig. 2. For determination of electrostatic field strength outside the dielectric body N_1 FCs are placed inside the body [2]. The FCs are placed at the cylindrical surface of radius $f_1 a$, where $f_1 < 1$.

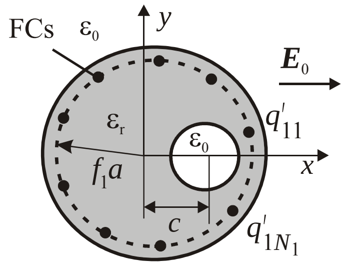

Fig. 2 - *First system of FCs*

Components of the electrostatic field vector outside the body are:

$$E_{1x} = E_0 + \sum_{j=1}^{N_1} \frac{q'_{1j}}{2\pi\varepsilon_0} \frac{(x - x_{1j})}{(x - x_{1j})^2 + (y - y_{1j})^2}; \tag{1}$$

$$E_{1y} = \sum_{j=1}^{N_1} \frac{q'_{1j}}{2\pi\varepsilon_0} \frac{(y - y_{1j})}{(x - x_{1j})^2 + (y - y_{1j})^2}, \tag{2}$$

where q'_{1j} ($j = 1,..., N_1$) is unknown line charge placed at the cylindrical surface and (x_{1j} , y_{1j}) is its position.

The second system (for determination of electrostatic field strength inside the dielectric body) is shown in Fig. 3. The FCs are placed at the cylindrical surfaces of radii $f_2 a$ and $f_3 b$, where $f_2 > 1$ and $0 < f_3 < 1$. With N_2 and N_3 the number of line charges in this system is denoted.

Components of the electrostatic field vector in this system are:

[1] University of Niš, Faculty of Electronic Engineering, Aleksandra Medvedeva 14, 18000 Nis, Serbia, E-mail: mika@elfak.ni.ac.yu
[2] University of Niš, Faculty of Electronic Engineering, Aleksandra Medvedeva 14, 18000 Nis, Serbia, E-mail: zora@elfak.ni.ac.yu
[3] University of Niš, Faculty of Electronic Engineering, Aleksandra Medvedeva 14, 18000 Nis, Serbia, E-mail: zlata@elfak.ni.ac.yu

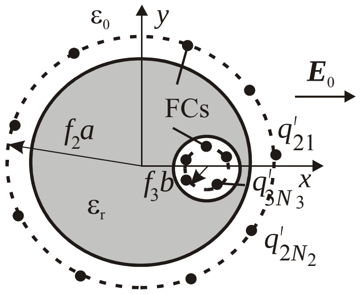

Fig. 3 - *Second system of FCs*

$$E_{2x}=\sum_{j=1}^{N_2}\frac{q'_{2j}}{2\pi\varepsilon}\frac{(x-x_{2j})}{(x-x_{2j})^2+(y-y_{2j})^2}+$$

$$+\sum_{k=1}^{N_3}\frac{q'_{3k}}{2\pi\varepsilon}\frac{(x-x_{3k})}{(x-x_{3k})^2+(y-y_{3k})^2}; \qquad (3)$$

$$E_{2y}=\sum_{j=1}^{N_2}\frac{q'_{2j}}{2\pi\varepsilon}\frac{(y-y_{2j})}{(x-x_{2j})^2+(y-y_{2j})^2}+$$

$$+\sum_{k=1}^{N_3}\frac{q'_{3k}}{2\pi\varepsilon}\frac{(y-y_{3k})}{(x-x_{3k})^2+(y-y_{3k})^2}, \qquad (4)$$

where q'_{2j} ($j=1,...,N_2$) and q'_{3j} ($k=1,...,N_3$) are unknown line charges placed at the cylindrical surfaces and (x_{2j} , y_{2j}) and (x_{3k} , y_{3k}) are their positions.

For determination of electrostatic field strength inside the cavity, FCs are placed inside the body, Fig. 4. The FCs are placed at the cylindrical surface of radius $f_4 b$, where $f_4>1$. The number of line charges in this system is N_4.

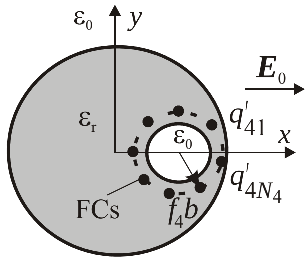

Fig. 4 - *Third system of FCs*

Components of the electrostatic field vector are:

$$E_{3x}=\sum_{j=1}^{N_4}\frac{q'_{4j}}{2\pi\varepsilon_0}\frac{(x-x_{4j})}{(x-x_{4j})^2+(y-y_{4j})^2}; \qquad (5)$$

$$E_{3y}=\sum_{j=1}^{N_4}\frac{q'_{4j}}{2\pi\varepsilon_0}\frac{(y-y_{4j})}{(x-x_{4j})^2+(y-y_{4j})^2}, \qquad (6)$$

where q'_{4j} ($j=1,...,N_4$) is unknown line charge placed at the cylindrical surface and (x_{4j} , y_{4j}) is its position.

NUMERICAL RESULTS

The dielectric body with circular cross-section and eccentric circular cavity is placed in transverse external electrostatic field, $\boldsymbol{E}_0=E_0\hat{\boldsymbol{x}}$. The electric field strength is E_0 = 100 V/m.

Input data are:

$f_1=0.9$, $f_2=1.2$, $f_3=0.9$, $f_4=1.1$, $b/a=0.25$, $c/a=0.45$, $\varepsilon_r=3$ and $N_1=N_2=N_3=N_4=N_e$.

Distributions of E_x and E_y electrostatic field vector components in the cavity along the x-axis are presented in Fig. 5 and Fig. 6.

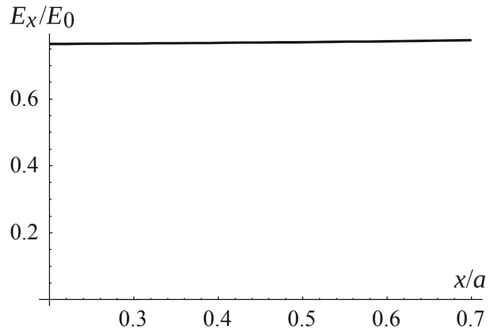

Fig. 5 - *Distribution of E_x electrostatic field component in the cavity along the x-axis*

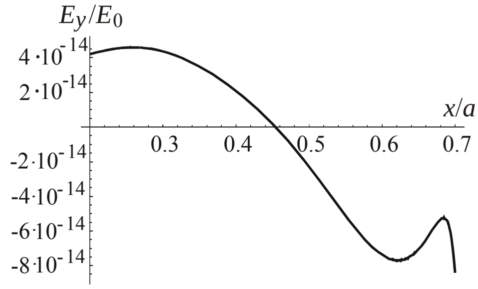

Fig. 6 - *Distribution of E_y electrostatic field component in the cavity along the x-axis*

From these figures it can be seen that the electrostatic field vector is homogeneous in the cavity. E_x component is almost constant and the value of E_y component is negligible.

In Table I a convergence of electrostatic field strength results for different number of FCs, N_e, is shown. The values are given in two characteristic points: centre of the body (0, 0) and centre of the cavity (0.45, 0).

Table I

Electrostatic field strength, E/E_0, in point $\mathrm{M}(x/a, y/a)$ for different number of fictitious charges

	E/E_0	
N_e	M(0,0)	M(0.45,0)
5	0.296062	0.219725
10	0.406967	0.458361
30	0.438597	0.732438
50	0.430652	0.764120
80	0.429240	0.768651
100	0.429184	0.768871
120	0.429177	0.768904

From this table a very good convergence of the results can be noticed. But, if $N_e>150$ the system of linear equations becomes ill conditioned, because the FCs are very close to each other, so the problem of division by zero appears. When $N_e = 120$, the results converge to 4 decimals.

An influence of parameter c/a on the electrostatic field strength in the centre of the cavity is shown in Fig. 7 [5].

Input data are:

$f_1 = 0.9$, $f_2 = 1.2$, $f_3 = 0.9$, $f_4 = 1.1$, $b/a = 0.25$, $\varepsilon_r = 3$ and $N_1 = N_2 = N_3 = N_4 = N_e$.

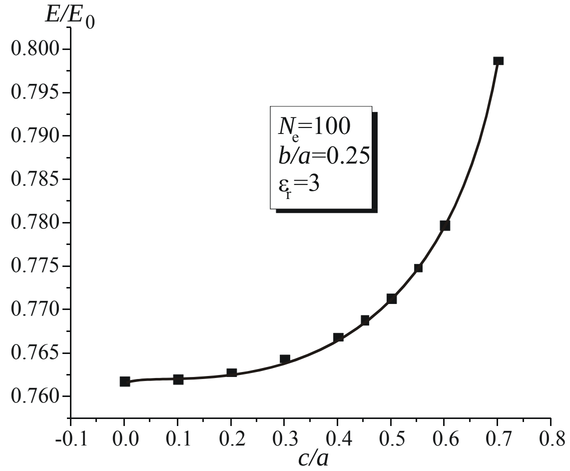

Fig. 7 - *Electrostatic field distribution in the centre of the cavity for different values of parameter* c/a

Increasing the value of eccentricity, c/a, better penetration of external electrostatic field, E/E_0, are obtained.

In Fig. 8 a dependence of the electrostatic field strength in the cavity for different values of relative permittivity, ε_r, is shown.

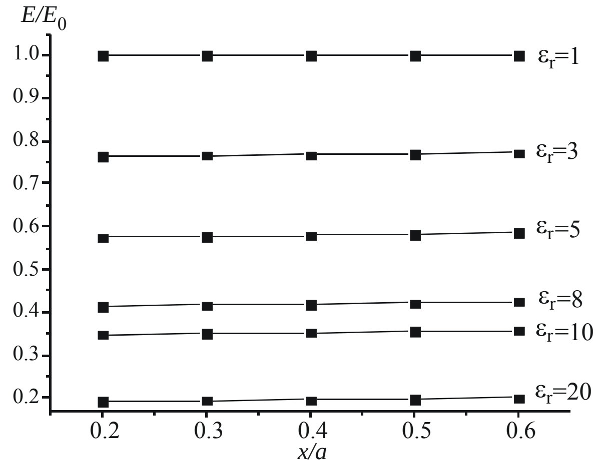

Fig. 8 - *Electrostatic field distribution in the cavity for different values of relative permittivity* ε_r

Also, a change of electrostatic field strength for different values of parameter ε_r in two characteristic points is given in Table II. Input data are:

$f_1 = 0.9$, $f_2 = 1.2$, $f_3 = 0.9$, $f_4 = 1.1$, $b/a = 0.25$, $c/a = 0.45$ and $N_1 = N_2 = N_3 = N_4 = N_e$.

An influence of parameter b/a on the electrostatic field strength was investigated in two characteristic points: centre of the body and centre of the cavity. These results are shown in Table III for $c/a = 0.0$ and in Table IV for $c/a = 0.45$.

Table II

Electrostatic field strength for different values of parameter ε_r *in two characteristic points*

ε_r	E/E_0 M(0,0)	E/E_0 M(0.45,0)
1	0.999968	0.99992
3	0.429184	0.768871
5	0.271740	0.580954
8	0.174913	0.420085
10	0.082062	0.353916
20	0.071941	0.197330

Input data are:

$f_1 = 0.9$, $f_2 = 1.2$, $f_3 = 0.9$, $f_4 = 1.1$, $\varepsilon_r = 3$ and $N_1 = N_2 = N_3 = N_4 = N_e$.

Table III

Electrostatic field strength for $c/a = 0.0$ *and different values of parameter* b/a

b/a	E/E_0 M(0,0)
0.1	0.751842
0.3	0.767228
0.5	0.799956
0.7	0.854641
0.9	0.940350

Table IV

Electrostatic field strength for $c/a = 0.45$ *and different values of parameter* b/a

b/a	E/E_0 M(0,0)	E/E_0 M(0.45,0)
0.05	0.497226	0.750706
0.15	0.474829	0.756659
0.25	0.429184	0.768871
0.35	0.359113	0.788059
0.45	0.263806	0.815649

From these tables it can be noticed that when parameter b/a increases, the electrostatic field strength increases, too.

Figs. 9-12 show the boundary condition satisfying for tangential components of electrostatic field strength at the separating surfaces: air - dielectric body and dielectric body – cavity. In ideal case these components should be equal.

Figs. 9-12 are given for different values of coefficients f_i $(i=1,2,3,4)$ and different number of FCs.

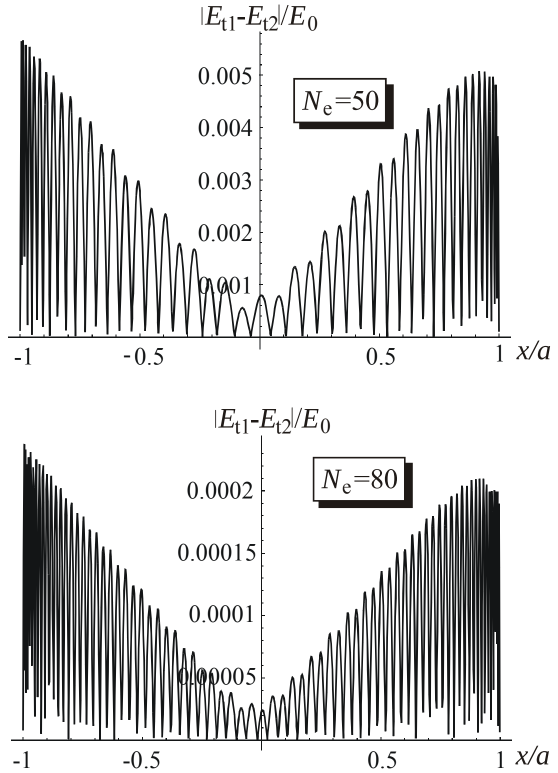

Fig. 9 - *Change of tangential component of electrostatic field at the separating surface air - dielectric body for*
$f_1=0.9$, $f_2=1.2$, $f_3=0.9$, $f_4=1.1$
and different number of fictitious charges

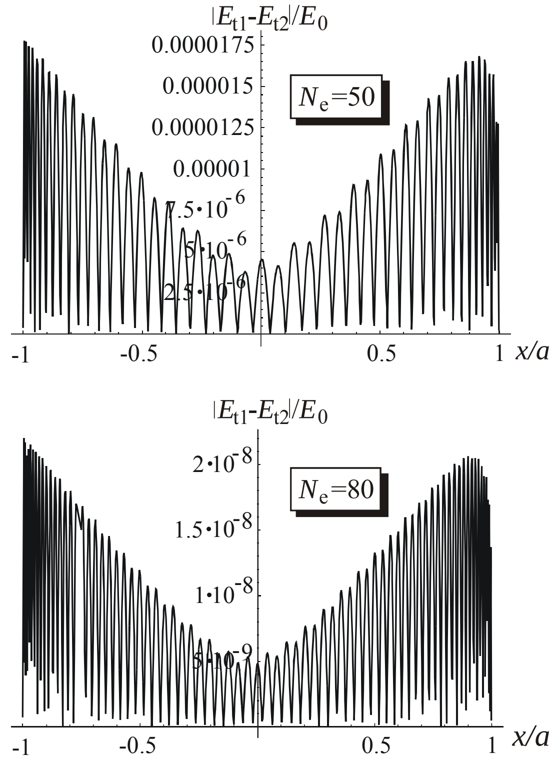

Fig. 10 - *Change of tangential component of electrostatic field at the separating surface air - dielectric body for*
$f_1=0.8$, $f_2=1.4$, $f_3=0.6$, $f_4=1.2$
and different number of fictitious charges

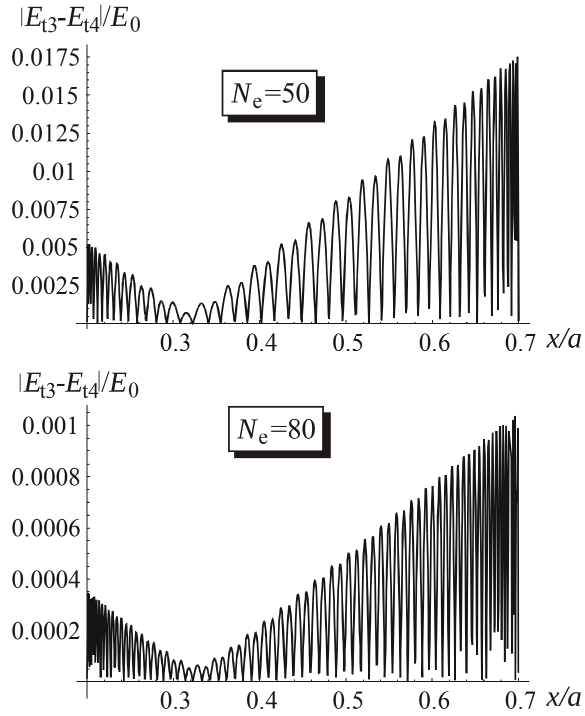

Fig. 11 - *Change of tangential component of electrostatic field at the separating surface dielectric body - cavity for*
$f_1=0.9$, $f_2=1.2$, $f_3=0.9$, $f_4=1.1$
and different number of fictitious charges

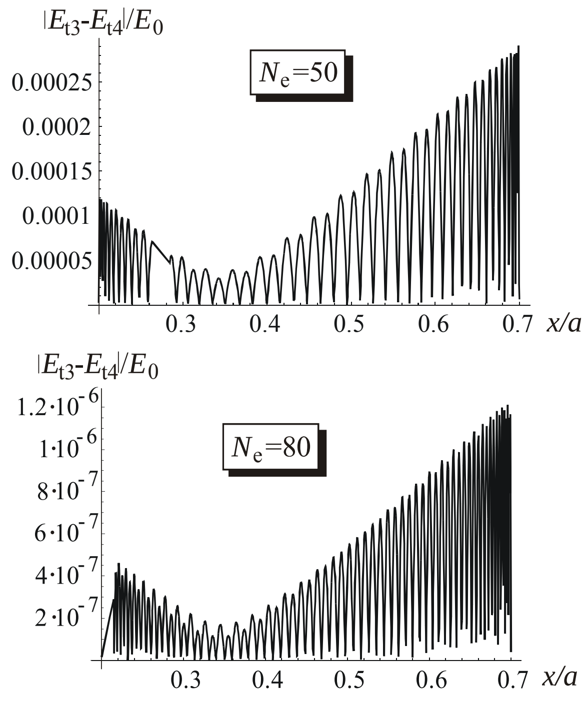

Fig. 12 - *Change of tangential component of electrostatic field at the separating surface dielectric body - cavity for*
$f_1=0.8$, $f_2=1.4$, $f_3=0.6$, $f_4=1.2$
and different number of fictitious charges

A good satisfaction of boundary condition is accomplished with increase of FCs number. However, better satisfaction of boundary condition is obtained when FCs are a little further from separating surface (Fig. 10 and Fig.

12). In that case the obtained accuracy has the order 10^{-6}-10^{-9}.

In Fig. 13, the results obtained by the CSM are compared to the results obtained by software for electrostatics problems solving (FEMLAB [7], femm [8] and QuickField [9]).

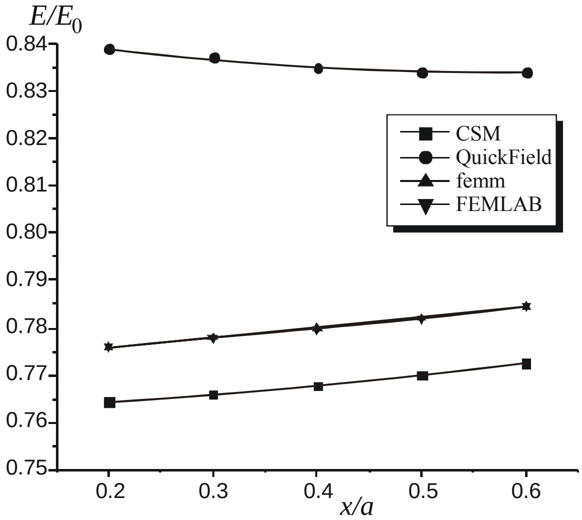

Fig. 13 - *Comparison of the results obtained by the CSM and results obtained by different software*

FEMLAB and femm software gave better results agreement than QuickField software. On the other hand, QuickField demands shorter time for these calculations.

CONCLUSION

In this paper, an application of CSM for calculation of electrostatic field strength inside the circular cavity of the cylindrical dielectric body is considered. The system is placed in the homogeneous transverse electrostatic field.

Theoretically, the precision of the solution depends on the number and position of FCs, i.e. a higher precision can be realized by increasing the number of FCs. However, if they are very close or too far from the cylindrical surface, the obtained error is higher.

A good satisfaction of boundary condition is accomplished with increase of number of FCs. Also, when the FCs are a little further from separating surface, better satisfaction of boundary condition is obtained.

Results, obtained by the CSM are compared to the results obtained by most commonly used software in Electromagnetic (QuickField, femm and FEMLAB). Certain disagreement with the CSM results is product of limited number of the nodes of Student's QuickField (max. 200 nodes).

It can be noticed a very good agreement CSM results with the results obtained by femm and FEMLAB software. FEMLAB, femm and QuickField software use Finite element method.

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Bojana Z. Nikolić was born in 1982 in Niš, Serbia. After finishing high school "Bora Stanković" Niš in 2001, she enrolled at the Faculty of Electronic Engineering, University of Niš. Her major of study is Telecommunications.

In the period from October 15, 2004 to December 15, 2004, she was chosen for a study visit at Technical University in Ilmenau, Germany. There she participated in the DAAD project "Academic Reconstruction of South Eastern Europe" and the cooperation between Niš and Ilmenau.

Currently, she is attending 5th year of regular studies with an average grade of 9.5.