Regular paper

PERMANENT MAGNET HOMOGENEOUSLY MAGNETIZED ALONG ITS AXES

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Abstract: The paper presents the magnetic field calculation of permanent magnet, homogeneously magnetized along its axes. Method used in the paper is based on a system of equivalent magnetic dipoles. The results that are obtained using this analytical method are compared with results obtained using program FEMLAB. Magnetic field and magnetic flux density distributions of permanent magnet are shown in the paper.

Keywords: Magnetic field, Permanent magnet, Magnetic dipole.

INTRODUCTION

To determine the magnetic field components in vicinity of permanent magnets, starts from presumption that magnetization, $\boldsymbol{M}$, of permanent magnet is known. The following methods are useful in practical calculation:

a) Method based on determining distribution of microscopic Ampere's current;

b) Method based on Poisson's and Laplace's equations, determining magnetic scalar potential; and

c) Method based on a system of equivalent magnetic dipoles.

Magnetic field inside and outside the permanent magnet, if magnetization of permanent magnet is known, can be calculated using equivalent system of volume and surface microscopic Ampere's currents, which are determined as

$$\boldsymbol{J}_{\mathrm{a}}(\boldsymbol{r}') = \mathrm{rot}\,\boldsymbol{M}(\boldsymbol{r}'), \text{ and} \tag{1}$$

$$\boldsymbol{J}_{\mathrm{sa}}(r') = \boldsymbol{M}(\boldsymbol{r}')\times\hat{\boldsymbol{n}}\,, \tag{2}$$

where $\hat{\boldsymbol{n}}$ is unit vector of outgoing normal (Fig.1).

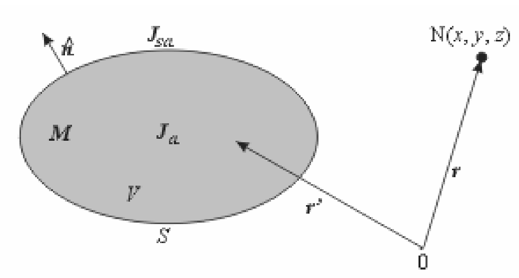

Fig.1 - *Permanent magnet*

These currents produce magnetic vector potential:

$$\boldsymbol{A}(\boldsymbol{r}) = \frac{\mu_0}{4\pi}\int_V \boldsymbol{J}_{\mathrm{a}}(\boldsymbol{r}')\frac{\mathrm{d}V'}{R} + \frac{\mu_0}{4\pi}\oint_S \boldsymbol{J}_{\mathrm{sa}}(\boldsymbol{r}')\frac{\mathrm{d}S'}{R}, \tag{3}$$

where $R = |\boldsymbol{r}-\boldsymbol{r}'|$.

Magnetic flux density is

$$\boldsymbol{B}(\boldsymbol{r}) = \mathrm{rot}\,\boldsymbol{A}(\boldsymbol{r})\,. \tag{4}$$

Inside a permanent magnet, magnetic field can be determined using relation

$$\boldsymbol{H} = \boldsymbol{B}/\mu_0 - \boldsymbol{M}\,. \tag{5}$$

Outside a permanent magnet, magnetic field can be determined using relation

$$\boldsymbol{H} = \boldsymbol{B}/\mu_0\,. \tag{6}$$

The second method is based on determining magnetic scalar potential φ_{m}. Inside a permanent magnet magnetic scalar potential satisfies Poisson's equation

$$\Delta\varphi_{\mathrm{m}} = \mathrm{div}\,\boldsymbol{M}\,. \tag{7}$$

Magnetic field vector can be presented as

$$\boldsymbol{H} = -\,\mathrm{grad}\,\varphi_{\mathrm{m}}\,. \tag{8}$$

Because outside a permanent magnet $\boldsymbol{M} = 0$, magnetic scalar potential, $\varphi_{\mathrm{m}0}$ satisfies Laplace's equation,

$$\Delta\varphi_{\mathrm{m}0} = 0\,, \tag{9}$$

where

$$\boldsymbol{H}_0 = -\,\mathrm{grad}\,\varphi_{\mathrm{m}0}\,. \tag{10}$$

The third method that is mentioned in the paper for magnetic field calculation is based on superposition of results obtained for elementary magnetic dipoles.

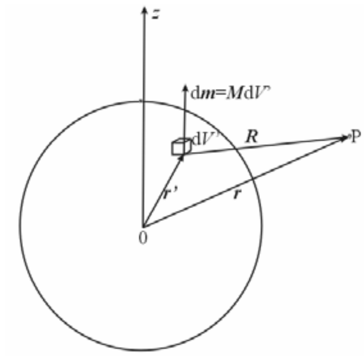

Fig.2 - *Elementary magnetic dipole*

Elementary magnetic dipole (Fig.2) has magnetic moment

$$\mathrm{d}\boldsymbol{m} = \boldsymbol{M}\,\mathrm{d}V'\,. \tag{11}$$

This magnetic moment produces, at field point P, elementary magnetic scalar potential

$$\mathrm{d}\,\varphi_{\mathrm{m}} = \frac{1}{4\pi}\frac{\boldsymbol{R}\,\mathrm{d}\boldsymbol{m}}{R^3} = \frac{1}{4\pi}\frac{\boldsymbol{R}\boldsymbol{M}}{R^3}\mathrm{d}V'\,, \tag{12}$$

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where $R=\left|\boldsymbol{r}-\boldsymbol{r}'\right|$ is distance from the point where the magnetic field is being calculated to elementary source, and $\boldsymbol{R}=\boldsymbol{r}-\boldsymbol{r}'$.

After integration magnetic scalar potential is obtained as

$$\varphi_{\mathrm{m}}=\frac{1}{4\pi}\int_V\frac{\boldsymbol{R}\,\mathrm{d}\boldsymbol{m}}{R^3}=\frac{1}{4\pi}\int_V\frac{\boldsymbol{RM}}{R^3}\mathrm{d}V'. \qquad (13)$$

PROBLEM DEFINITION

Permanent magnet that is observed in the paper is homogeneously magnetized along its axes. This is magnetic circuit that is made of ferromagnetic material and it consists of four parts. Each of these parts is magnetized in direction shown in the Fig.3. Dimensions of the permanent magnet are also presented in the Fig.3.

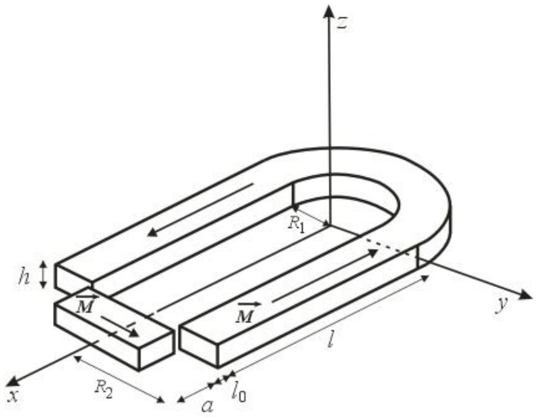

Fig.3 – *Permanent magnet*

Outside the permanent magnet magnetic scalar potential, at field point $\mathrm{P}(x,y,z)$, can be determined using superposition of results obtained for each magnetized part,

$$\varphi_{\mathrm{m}}=\varphi_{\mathrm{m}1}+\varphi_{\mathrm{m}2}+\varphi_{\mathrm{m}3}+\varphi_{\mathrm{m}4}, \qquad (14)$$

where $\varphi_{\mathrm{m}1}$ and $\varphi_{\mathrm{m}2}$ are magnetic scalar potentials of two ends that are magnetized in different direction, $\varphi_{\mathrm{m}3}$ is magnetic scalar potentials produced by the part which is magnetized in angular direction and $\varphi_{\mathrm{m}4}$ is magnetic scalar potentials of fourth part.

These magnetic scalar potentials can be determined using the expression (13), where

$$R=\left|\boldsymbol{r}-\boldsymbol{r}'\right|=\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}, \quad \text{and} \qquad (15)$$

$$\boldsymbol{R}=\boldsymbol{r}-\boldsymbol{r}'=(x-x')\hat{\boldsymbol{x}}+(y-y')\hat{\boldsymbol{y}}+(z-z')\hat{\boldsymbol{z}}. \qquad (16)$$

The first part of the magnetic circuit is homogeneously magnetized in negative direction of x-axis,

$$\boldsymbol{M}=-M\hat{x}. \qquad (17)$$

Scalar product is $\boldsymbol{RM}=-M(x-x')$. (18)

Substituting expressions (15) and (18) in (13), magnetic scalar potential $\varphi_{\mathrm{m}1}$ is obtained

$$\varphi_{\mathrm{m}1}=-\frac{M}{4\pi}\int_{-\frac{h}{2}}^{\frac{h}{2}}\int_{R_1}^{R_2}\int_0^l\frac{(x-x')\,\mathrm{d}x'\,\mathrm{d}y'\,\mathrm{d}z'}{\left((x-x')^2+(y-y')^2+(z-z')^2\right)^{3/2}} \qquad (19)$$

The second part of the magnetic circuit is homogeneously magnetized in positive direction of x-axis,

$$\boldsymbol{M}=M\hat{x}. \qquad (20)$$

Scalar product is $\boldsymbol{RM}=M(x-x')$. (21)

Substituting expressions (15) and (21) in (13), magnetic scalar potential $\varphi_{\mathrm{m}2}$ can be calculated

$$\varphi_{\mathrm{m}2}=\frac{M}{4\pi}\int_{-\frac{h}{2}}^{\frac{h}{2}}\int_{-R_2}^{-R_1}\int_0^l\frac{(x-x')\,\mathrm{d}x'\,\mathrm{d}y'\,\mathrm{d}z'}{\left((x-x')^2+(y-y')^2+(z-z')^2\right)^{3/2}} \qquad (22)$$

The third part of the magnetic circuit is homogeneously magnetized in angular direction

$$\boldsymbol{M}=M\hat{\theta}. \qquad (23)$$

Relations between coordinates x, y, z and cylindrical coordinates r,θ and z are

$$x=r\cos\theta,\ y=r\sin\theta \text{ and } z=z. \qquad (24)$$

Using these relations, distance from the point where the magnetic field is being calculated to elementary source, can be presented as

$$R=\sqrt{r^2+r'^2-2rr'\cos(\theta-\theta')+(z-z')^2}. \qquad (25)$$

As magnetization has only angular component θ, scalar product $\boldsymbol{RM}$ is obtained as

$$\boldsymbol{RM}=\left[(x-x')\hat{\boldsymbol{x}}+(y-y')\hat{\boldsymbol{y}}+(z-z')\hat{\boldsymbol{z}}\right]M\,\hat{\theta}. \qquad (26)$$

Relation between unit vectors $\hat{\boldsymbol{x}}$, $\hat{\boldsymbol{y}}$, $\hat{\boldsymbol{z}}$ and $\hat{\theta}$ is

$$\hat{\theta}=\frac{1}{r}\left(\frac{\partial x}{\partial\theta}\hat{\boldsymbol{x}}+\frac{\partial y}{\partial\theta}\hat{\boldsymbol{y}}+\frac{\partial z}{\partial\theta}\hat{\boldsymbol{z}}\right), \qquad (27)$$

and the following expressions are also satisfied

$$\hat{\theta}\,\hat{\boldsymbol{x}}=\frac{1}{r}\frac{\partial x}{\partial\theta},\ \hat{\theta}\,\hat{\boldsymbol{y}}=\frac{1}{r}\frac{\partial y}{\partial\theta},\ \hat{\theta}\,\hat{\boldsymbol{z}}=\frac{1}{r}\frac{\partial z}{\partial\theta}. \qquad (28)$$

Using relations (24) and (28), the following relations are obtained

$$\hat{\theta}\,\hat{\boldsymbol{x}}=-\sin\theta,\ \hat{\theta}\,\hat{\boldsymbol{y}}=\cos\theta,\ \hat{\theta}\,\hat{\boldsymbol{z}}=0, \qquad (29)$$

and scalar product can be presented as

$$\boldsymbol{RM}=Mr'\sin(\theta-\theta'). \qquad (30)$$

Substituting expressions (25) and (30) in (13), magnetic scalar potential is calculated as

$$\varphi_{\mathrm{m}3}=\frac{M}{4\pi}\int_{R_1}^{R_2}\int_{-\frac{h}{2}}^{\frac{h}{2}}\int_0^{\pi}\frac{r'\sin(\theta-\theta')\,\mathrm{d}r'\,\mathrm{d}z'\,\mathrm{d}\theta'}{\left[r^2+r'^2-2rr'\cos(\theta-\theta')+(z-z')^2\right]^{3/2}}$$

(31)

The fourth part of the permanent magnet is homogeneously magnetized in positive direction of y-axis,

$$\boldsymbol{M} = M\hat{y}\,. \tag{32}$$

Scalar product is $$\boldsymbol{RM} = M(y-y')\,. \tag{33}$$

Substituting expressions (15) and (33) in (13), magnetic scalar potential φ_{m4} is obtained

$$\varphi_{m4} = \frac{M}{4\pi}\int_{-\frac{h}{2}}^{\frac{h}{2}}\int_{h-R_1}^{R_1}\int_{l+l_0}^{l+l_0+a}\frac{(y-y')\,\mathrm{d}x'\,\mathrm{d}y'\,\mathrm{d}z'}{\left((x-x')^2+(y-y')^2+(z-z')^2\right)^{3/2}}$$

(34)

The solutions of integrals presented in the expressions (19), (22), (31) and (34) are very complex. Because of that the expression for magnetic scalar potential (14) is very large and it won't be shown in the paper, but it is used for determining the components of magnetic field.

Magnetic field vector can be expressed as

$$\boldsymbol{H} = -\operatorname{grad}\varphi_m\,, \tag{35}$$

therefore its components are

$$H_x = -\frac{\partial\varphi_m}{\partial x}\,,\; H_y = -\frac{\partial\varphi_m}{\partial y}\,,\; H_z = -\frac{\partial\varphi_m}{\partial z}\,. \tag{36}$$

NUMERICAL RESULTS

Distribution of magnetic flux density outside the permanent magnet is presented in the Fig.4.

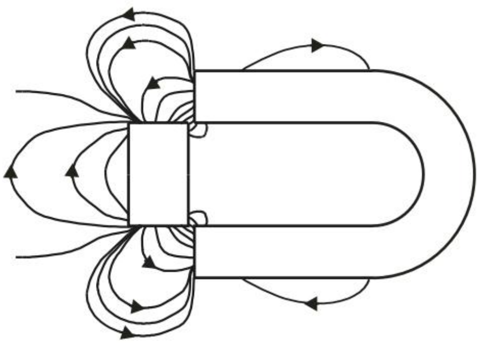

Fig.4 – *Distribution of magnetic flux density*

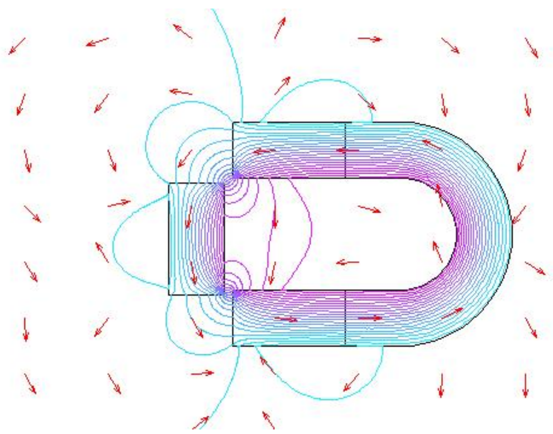

Fig.5 – *Distribution of magnetic flux density (FEMLAB)*

Magnetic flux density is obtained using the analytical method for the following dimension of permanent magnet $R_1/h = 2$, $R_2/h = 4$, $a/h = 2$, $l/h = 6$ and $l_0/h = 0.2$.

The Fig.5 presents distribution of magnetic flux density (arrow) and magnetic potential (contour) obtained using FEMLAB software.

Comparing these figures, the excellent agreement between analytical method results and FEMLAB results is evident.

Table I

Normalized magnetic field values along the direction $y = 0, z = 0$

x/l	H/M
0.1	1.223270
0.2	0.515419
0.3	0.267753
0.4	0.150165
0.5	0.087647
0.6	0.051969
0.7	0.030798
0.8	0.019224
0.9	0.016823
1.0	0.023926

Table II

Normalized magnetic field values along the direction $x/l = 1.0167, z = 0$

y/l	H/M
0.1	0.028094
0.2	0.054448
0.3	0.257539
0.4	0.354298
0.5	0.195708
0.6	0.184541
0.7	0.182569
0.8	0.042942
0.9	0.015348
1.0	0.007758
1.1	0.006290
1.2	0.006057
1.3	0.005785

In the Table I and Table II, magnetic field values along characteristic directions, for mentioned dimensions of permanent magnet, are presented.

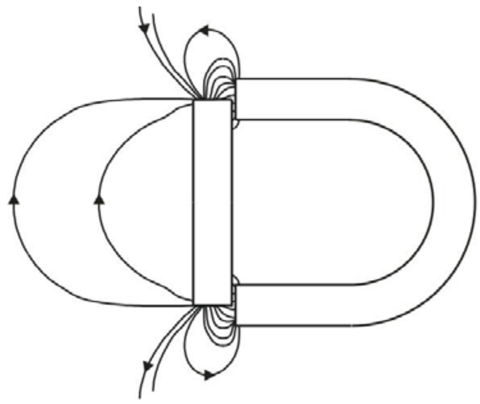

Fig.6 – *Distribution of magnetic flux density*

When the length of the part that is magnetized along y-axis, is equal to $R_1 + R_2$, distribution of magnetic flux density outside the permanent magnet is presented in the Fig.6. It is obtained using the analytical method, for the following dimension of permanent magnet $R_1/h = 4$, $R_2/h = 6$, $a/h = 2$, $l/h = 6$ and $l_0/h = 0.2$.

The Fig.7 presents distribution of magnetic flux density (arrow) and magnetic potential (contour) obtained using FEMLAB software.

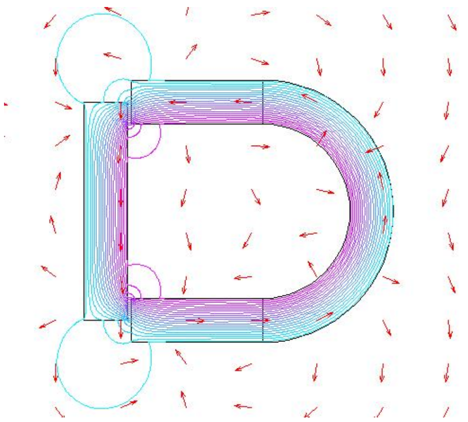

Fig.7 – *Distribution of magnetic flux density (FEMLAB)*

Comparing Fig.6 and Fig.7 the results obtained by analytical method are satisfactory confirmed using FEMLAB software.

In the Table III and Table IV, magnetic field values along characteristic direction, for mentioned dimensions of permanent magnet, are presented.

Table III

Normalized magnetic field values along the direction $y = 0, z = 0$

x/l	H/M
0.1	0.438976
0.2	0.210030
0.3	0.128901
0.4	0.085889
0.5	0.059395
0.6	0.041883
0.7	0.029927
0.8	0.021712
0.9	0.016179
1.0	0.012612

Table IV

Normalized magnetic field values along the direction $x/l = 1.0167, z = 0$

y/l	H/M
0.1	0.028094
0.2	0.054448
0.3	0.257539
0.4	0.354298
0.5	0.195708
0.6	0.184541
0.7	0.182569
0.8	0.042942
0.9	0.015348
1.0	0.007758
1.1	0.006290
1.2	0.006057
1.3	0.005785

(continue)

CONCLUSION

Permanent magnet, homogeneously magnetized along its axes is observed in the paper. Method that is used for magnetic field determination is based on superposition of results that are obtained for elementary magnetic dipoles. The tables with magnetic field values, in different points, in vicinity of permanent magnet, are shown. Magnetic flux density distribution of permanent magnet is also presented in the paper. Magnetic field lines have the same form and the same direction as magnetic flux density lines, outside the magnet. Results obtained by analytical method are satisfactory confirmed using FEMLAB software.

REFERENCES

[1] Textbook of Electromagnetics, Edited by Hermann Uhlmann (Veličković D. M., Chapter 3.2: Magnetostatic field), Technische Universitaet Ilmenau.

[2] A. N. Mladenović, S. R. Aleksić: "Rod shaped permanent magnet", 48. Conference ETRAN 2004, Čačak, 6th-11th Jun, Proceedings of full papers, pp. 233-236. (In Serbian)

[3] A. N. Mladenović, Slavoljub R. Aleksić: Toroidal shaped permanent magnet with air gap, International Conference on Applied Electromagnetics PES2005, Niš 23rd-25th May 2005, Proceedings of Extended Abstracts, pp. 23-24.

[4] Ana N. Mladenović, Slavoljub R. Aleksić:" Methods for magnetic field calculation", 11th International Conference on Electrical Machines, Drives and Power Systems ELMA 2005, Sofia, Bulgaria, 15-16 September 2005, Vol.2, pp. 350-354.

[5] Ana N. Mladenović: "Toroidal shaped permanent magnet with two air gaps", International PhD-Seminar "Numerical Field Computation and Optimization in Electrical Engineering", Proceedings of Full Papers, Ohrid, Macedonia, 20-25 September 2005, pp.153-158.

Ana N. Mladenović was born in Niš, Serbia on July 29, 1977. In 1996 she enrolled in the Faculty of Electronic Engineering, University of Niš, Serbia and Montenegro, with the Computer Science as major. She graduated in 2003 with the final thesis in Electromagnetic. She enrolled in the post-graduate studies at the same faculty at the Department of Theoretical Electrical Engineering, in 2004.

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