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SIMPLIFICATIONS OF SYNCHRONOUS MACHINE PARAMETERS IN STABILITY STUDIES AND REACTIVE CAPABILITY LIMITS

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Abstract: In this paper, briefly, the problem of power sytem stability is considered. After this introduction, some simplifications required for the representation of synchronous machines in stability studies are discussed. Also considered are various degrees of approximations that can be made to simplify the machine model, minimizing data requirements and computational effort.

Keywords: Synchronous machine parameters, Stability, Reactive capability limits, Leakage flux.

INTRODUCTION

Power system stability is a single problem; however, it is impractical to study it as such. Instability of a power system can take different forms and can be influenced by a wide range of factors. Analysis of stability problems, identification of essential factors that contribute to instability, and formation of methods of improving stable operation are greatly facilitated by classification of stability into appropriate categories. These are based on the following considerations:

- the physical nature of the resulting instability;
- the size of the disturbance considered;
- the devices, processes, and time span that must be taken into consideration in order to determine stability; and
- the most appropriate method of calculation and prediction of stability.

Power system stability can be defined as ability to remain in operating equilibrium (equilibrium between opposing forces), and be classified into:

- angle stability (ability to maintain synchronism, torque balance of synchronous machines)
- voltage stability (ability to maintain steady acceptable voltage, reactive power balance)

While classification of power system stability is an effective and convenient means to deal with the complexities of the problem, the overall stability of the system should always be kept in mind. Solutions to stability problems of one category should not be at the expense of another. It is essential to look at all aspects of the stability phenomena and at each aspect from more than one view point.

SIMPLIFICATIONS ESSENTIAL FOR LARGE-SCALE STUDIES

Concerning per unit stator voltage equations [1]:

$$e_d = p\psi_d - \psi_q\omega_r - R_a i_d \qquad (1)$$

$$e_q = p\psi_q + \psi_d\omega_r - R_a i_q \qquad (2)$$

it is necessary to neglect the following from above equations, for reasons of stability analysis of large systems:

- the transformer voltage terms, $p\psi_d$ and $p\psi_q$;
- the effects of speed variations.

The reasons for and the effects of these simplifications are discussed below.

Neglect of stator $p\psi$ **terms**

The $p\psi_d$ and $p\psi_q$ terms represent the stator transients. With these terms neglected, the stator quantities contain only fundamental frequency components and the stator voltage equations appear as algebraic equations. This allows the use of steady-state relationships for representing the interconnecting transmission network.

Figure 1 shows the system studied. It consists of a salient-pole generator connected to an infinite bus through two transmission lines. The disturbance applied is a three-phase short-circuit at the sending end of one of the lines, cleared in 0.09s by isolating the faulted circuit. The responses of generator variables computed with and without inclusion of the stator $p\psi$ terms are compared in Figures 2, 3 and 4 [1].

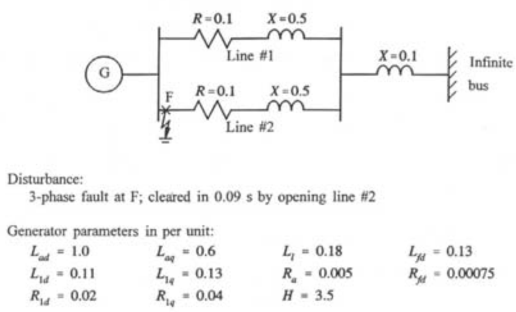

Figure 1 – *System configuration and parameters*

When the stator $p\psi$ terms are omitted, we see from Figure 2 that i_d and i_q have only unidirectional components; these correspond to the fundamental frequency component of phase currents. The resulting air-gap torque is unidirectional and small in magnitude; it is due to the stator resistance losses.

On the other hand, when the $p\psi$ terms are included, i_d and i_q contain fundamental frequency (50 Hz) components, which correspond to the dc offset in the phase currents. These in turn result in the following components of air-gap torque:

- a fundamental frequency oscillatory component, due to interaction with the rotor field;
- a unidirectional component, due to rotor resistance losses caused by the fundamental frequency currents induced in the rotor.

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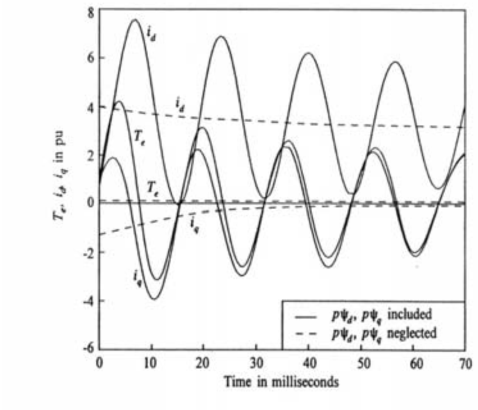

Figure 2 – *Effect of neglecting stator transients on air-gap torque and d-q components of stator currents*

The unidirectional component of torque, due to rotor resistive losses, can be quite high and has a braking effect. Therefore, it is referred to as the *dc braking torque*; its effect of the oscillatory component is to decelerate the rotor during the first half cycle and to accelerate it to near its initial speed during the second half cycle, and so on during subsequent cycles. The net effect of the oscillatory torque is therefore a reduction of the mean speed of the rotor [2]. The overall effect of these two components for a close-up simultaneous three-fase fault could be large enough to initially cause retardation of the rotor or a *back swing*. This could have a significant beneficial effect on system stability as seen from plots of speed deviation and rotor angle in Figures 3 and 4.

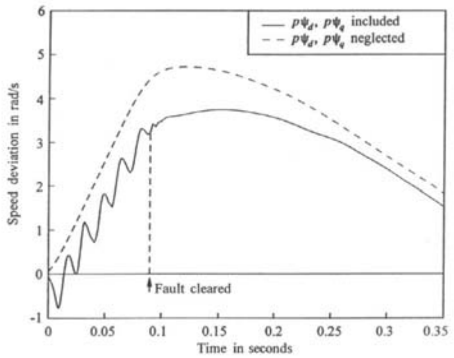

Figure 3 – *Effect of neglecting stator transients on speed deviations*

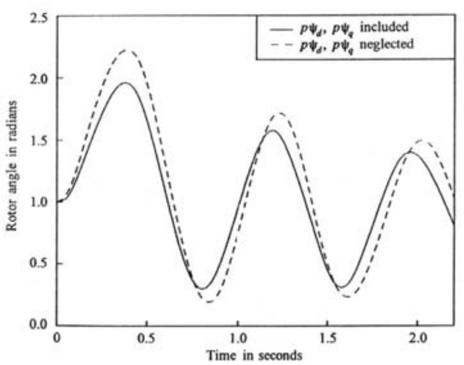

Figure 4 – *Effect of neglecting stator transients on rotor angle swings*

Since, for large-scale stability studies, it is necessary to neglect the $p\psi$ terms, the effects of the unidirectional (dc) braking torque and the oscillatory torque may be estimated and included in the calculations.

With the stator transients neglected, the per unit stator voltage Equations 1 and 2 appear as algebraic equations:

$$e_d = -\psi_q \omega_r - R_a i_d \quad (3)$$

$$e_q = \psi_d \omega_r - R_a i_q \quad (4)$$

Neglecting the effect of speed variations on stator voltages

Another simplifying assumption normaly made is that the per unit value of ω_r is equal to 1.0 in the stator voltage equations. This is not the same as saying that speed is constant; it assumes that speed changes are small and do not have a significant effect on the voltage.

The assumption of per unit ω_r=1.0 (i.e., ω_r=ω_o rad/s) in the stator voltage equations does not contribute to computational simplicity in itself. The primary reason for making this assumption is that it counterbalances the effect of neglecting $p\psi_d$, $p\psi_q$ terms so far as the low-frequency rotor oscillations are concerned.

With per unit ω_r=1.0, the stator voltage equations (1 and 2) reduce to

$$e_d = -\psi_d - R_a i_d \quad (5)$$

$$e_q = \psi_d - R_a i_q \quad (6)$$

Thus, per unit rotor voltage, stator flux linkage, and rotor flux linkage equations below [1] remain the same, as same as per unit air-gap torque expression:

Per unit rotor voltage equations:

$$e_{fd} = p\psi_{fd} + R_{fd} i_{fd} \quad (7)$$

$$0 = p\psi_{1d} + R_{1d} i_{1d} \quad (8)$$

$$0 = p\psi_{1q} + R_{1q} i_{1q} \quad (9)$$

$$0 = p\psi_{2q} + R_{2q} i_{2q} \quad (10)$$

Per unit stator flux linkage equations:

$$\psi_d = -(L_{ad} + L_l) i_d + L_{ad} i_{fd} + L_{ad} i_{1d} \quad (11)$$

$$\psi_q = -(L_{aq} + L_l) i_q + L_{aq} i_{1q} + L_{aq} i_{2q} \quad (12)$$

$$\psi_0 = -L_0 i_0 \quad (13)$$

Per unit rotor flux linkage equations:

$$\psi_{fd} = L_{ffd} i_{fd} + L_{f1d} i_{1d} - L_{ad} i_d \quad (14)$$

$$\psi_{1d} = L_{f1d} i_{fd} + L_{11d} i_{1d} - L_{ad} i_d \quad (15)$$

$$\psi_{1q} = L_{11q} i_{1q} + L_{aq} i_{2q} - L_{aq} i_q \quad (16)$$

$$\psi_{2q} = L_{aq} i_{1q} + L_{22q} i_{2q} - L_{aq} i_q \quad (17)$$

Per unit air-gap torque:

$$T_e = \psi_d i_q - \psi_q i_d \quad (18)$$

In writing Equations 16 and 17, we have assumed that the per unit mutual inductance L_{12q} is equal to L_{aq}. This implies that the stator and rotor circuits in the q-axis all link a single mutual flux represented by L_{aq}. This is acceptable because the rotor circuits represent the overall rotor body effects, and actual windings with physically measurable voltages and currents do not exist.

SIMPLIFIED MODEL WITH AMORTISSEURS NEGLECTED

The first order of simplification to the synchronous machine model is to neglect the amortisseur effects. This minimizes data requirements since the machine parameters related to the amortisseurs are often not readily available. In addition, it may contribute to reduction in computational effort by reducing the order of the model and allowing larger integration steps in time-domain simulations.

With the amortisseurs neglected, the stator voltage equations 5 and 6 are unchanged. The remaining equations (7 to 17) simplify as follows.

Flux linkages:

$$\psi_d = -L_d i_d + L_{ad} i_{fd} \tag{19}$$

$$\psi_q = -L_q i_q \tag{20}$$

$$\psi_{fd} = -L_{ad} i_d + L_{ffd} i_{fd} \tag{21}$$

Rotor voltage:

$$e_{fd} = p\psi_{fd} + R_{fd} i_{fd} \tag{22}$$

or

$$p\psi_{fd} = e_{fd} - R_{fd} i_{fd} \tag{23}$$

Equation 23 is now the only differential equation associated with the electrical characteristics of the machine. In the above equations all quantities, including time, are in per unit.

CONSTANT FLUX LINKAGE MODEL

Classical model

For studies in which the period of analysis is small in comparison to T'_{d0} (open circuit time-constant), the machine model of previous Section is often simplified by assuming E'_q (or ψ_{fd}) constant throughout the study period. This assumption eliminates the only differential equation associated with the electrical characteristics of the machine.

A further approximation, which simplifies the machine model significantly, is to ignore transient saliency by assuming $X'_d = X'_q$, and to assume that the flux linkage ψ_{1q} (associated with the *q*-axis rotor circuit corresponding to X'_q) also remains constant. With these assumptions, as shown below, the voltage behind the transient impedance $R_a + jX'_d$ has a constant magnitude.

The *d*- and *q*-axis equivalent circuits with only one circuit in each axis are shown in Figure 5 [1].

The per unit flux linkages identified in the *d*-axis are given by

$$\psi_{ad} = -L_{ad} i_d + L_{ad} i_{fd} \tag{24}$$

$$\psi_d = \psi_{ad} - L_l i_d \tag{25}$$

$$\psi_{fd} = \psi_{ad} + L_{fd} i_{fd} \tag{26}$$

From equation 26

$$i_{fd} = \frac{\psi_{fd} - \psi_{ad}}{L_{fd}} \tag{27}$$

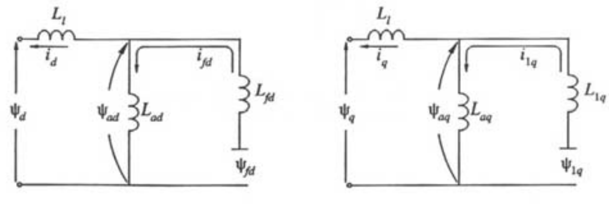

Figure 5 – *The d- and q-axis equivalent circuits with one rotor circuit in each axis*

Substituting in Equation 24 gives

$$\psi_{ad} = -L_{ad} i_d + \frac{L_{ad}}{L_{fd}}\left(\psi_{fd} - \psi_{ad}\right) \tag{28}$$

Rearranging to express ψ_{ad} in terms of ψ_{fd}, we find

$$\psi_{ad} = L'_{ad}\left(-i_d + \frac{\psi_{fd}}{L_{fd}}\right) \tag{29}$$

where

$$L'_{ad} = \frac{1}{\dfrac{1}{L_{ad}} + \dfrac{1}{L_{fd}}} = L'_d - L_l \tag{30}$$

Similarly, for the *q*-axis

$$\psi_{aq} = L'_{aq}\left(-i_q + \frac{\psi_{1q}}{L_{1q}}\right) \tag{31}$$

where

$$L_{aq} = L'_d - L_l$$

From Equation 3, the *d*-axis stator voltage is given by

$$e_d = -R_a i_d - \omega\psi_q = -R_a i_d + \omega\left(L_l i_q - \psi_{aq}\right)$$

where $\omega = \omega_r = \omega_0 = 1.0$ pu. Substituting for ψ_{aq} from Equation 31 gives

$$\begin{aligned} e_d &= -R_a i_d + \omega L_l i_q - \omega L'_{aq}\left(-i_q + \frac{\psi_{1q}}{L_{1q}}\right) \\ &= -R_a i_d + \omega\left(L_l + L'_{aq}\right) i_q - \omega L'_{aq}\left(\frac{\psi_{1q}}{L_{1q}}\right) \qquad (32) \\ &= -R_a i_d + X'_q i_q + E'_d \end{aligned}$$

where

$$E_d^{'} = -\omega L_{aq}^{'}\left(\frac{\psi_{1q}}{L_{1q}}\right) \tag{33}$$

Similarly, the q-axis stator voltage is given by

$$e_q = -R_a i_q - X_d^{'} i_d + E_q^{'} \tag{34}$$

where

$$E_q^{'} = \omega L_{ad}^{'}\left(\frac{\psi_{fd}}{L_{fd}}\right) \tag{35}$$

With transient saliency neglected ($X_d^{'} = X_q^{'}$), the stator terminal voltage is

$$\begin{aligned} e_d + je_q &= \left(E_d^{'} + jE_q^{'}\right) - R_a\left(i_d + ji_q\right) + X_d^{'}\left(i_q - ji_d\right) \\ &= \left(E_d^{'} + jE_q^{'}\right) - R_a\left(i_d + ji_q\right) - jX_d^{'}\left(i_d + ji_q\right) \end{aligned}$$

Using phasor notation, we have

$$\widetilde{E}_t = \widetilde{E}^{'} - \left(R_a + jX_d^{'}\right)\widetilde{I}_t \tag{36}$$

where

$$\widetilde{E}^{'} = E_d^{'} + jE_q^{'} = L_{ad}^{'}\left(-\frac{\psi_{1q}}{L_{1q}} + j\frac{\psi_{fd}}{L_{fd}}\right)$$

The corresponding equivalent is shown in Figure 6.

With rotor flux linkages (ψ_{fd} and ψ_{1q}) constant, $E_d^{'}$ and $E_q^{'}$ are constant. Therefore, the magnitude of $E^{'}$ is constant. As the rotor speed changes, the d- and q-axes move with respect to any general reference coordinate system whose R-I axes rotate at synchronous speed, as shown in Figure 7. Hence, the components $E_R^{'}$ and $E_I^{'}$ change.

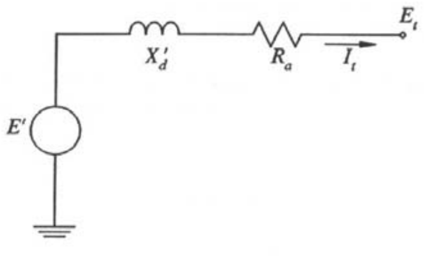

Figure 6 – *Simplified transient model*

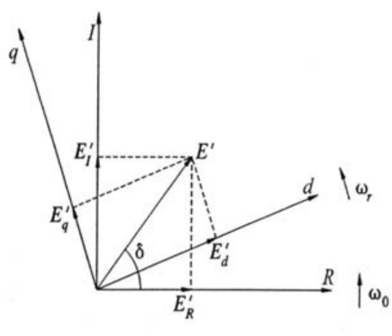

Figure 7 – *The R-I and d-q coordinate systems*

The magnitude of $E^{'}$ can be determined by computing its pre-disturbance value.

$$\widetilde{E}^{'} = \widetilde{E}_{t0} + \left(R_a + jX_d^{'}\right)\widetilde{I}_{t0} \tag{37}$$

Its magnitude is then assumed to remain constant throughout the study period. Since R_a is small, it is usuall to neglect it.

With the components $E_d^{'}$ and $E_q^{'}$ each having a constant magnitude, $E^{'}$ will have constant orientation with respect to d- and q-axes, as the rotor speed changes. Therefore, the angle of $E^{'}$ with respect to synchronously rotating reference axes (R-I) can be used as a measure of the rotor angle.

This model offers considerable computational simplicity; it allows the transient electrical performance of the machine to be represented by a simple voltage source of fixed magnitude behind an effective reactance. It is commonly reffered to as the *classical model,* since it was used extensively in early stability studies.

REACTIVE CAPABILITY LIMITS

In voltage stability and long-term stability studies, it is important to consider the reactive capability limits of synchronous machines.

Synchronous generators are rated in terms of the maximum MVA output at a specified voltage and power factor (usually 0.85 or 0.9 lagging) which they can carry continuously without overheating. The active power output is limited by the prime mover capability to a value within the MVA rating. The continous reactive power output capability is limited by three considerations: armature current limit, field current limit, and end region heating limit.

Armature current limit

The armature current results in an RI^2 power loss, and the energy associated with this loss must be removed so as to limit the increase in temperature of the conductor and its immediate environment. Therefore, one of the limitations on generator rating is the maximum current that can be carried by the armature without exceeding the heating limitations.

Field current limit

Because of the heat resulting from the $R_{fd} i_{fd}^2$ power loss, the field current imposes a second limit on the operation of the generator.

End region heating limit

The localized heating in the end region of the armature imposes a third limit on the operation of a synchronous machine. As explained below, this limit affects the capability of the machine in the underexcited condition.

Figure 8 is a schematic of the end-turn region of a generator. The end-turn leakage flux, as shown in the figure, enters and leaves in a direction perpendicular (axial) to the stator laminations. This causes eddy currents in the laminations, resulting in localized heating in the end region. The high field currents corresponding to the overexcited condition keep the retaining ring saturated, so that end leakage flux is small. However, in the underexcited region the field current is low and the retaining ring is not saturated; this permits an increase in armature and leakage flux [3]. Also, in the underexcited condition, the flux produced by the armature currents adds to the flux produced by the field current; therefore, the end-turn flux enhances the axial flux in the end region and the resulting heating effect may severely limit the generator output, particularly in the case of a round rotor machine.

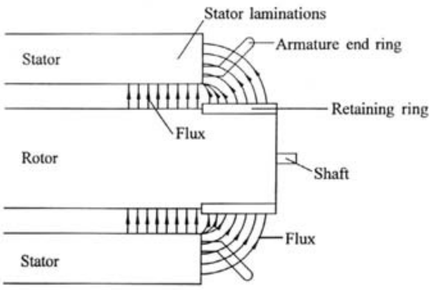

Figure 8 – *Sectional view of end region of a generator*

V curves and compounding curves

The curve showing the relation between armature and field current at a constant terminal voltage and with constant active power output is known as a *V curve*.

Figure 9 shows solid curves shown for three values of *P* (0.5, 0.7, 0.85 pu). The dashed lines are loci of constant power factor and are known as *compounding curves*. Each of these curves shows how field current has to vary in order to maintain a constant power factor.

Also shown in Figure 9 are the reactive capability limits for one value of hydrogen pressure (45 PSIG). The three segments AB, BC and CD correspond to the field current limit, armature current limit, and end region heating limit, respectively. Since the characteristics shown in Figure 9 apply at rated stator terminal voltage, the per unit values of armature current and apparent power output are equal, and hence both are shown along the ordinate. The field current plotted along the abscissa is the normalized value, with 1.0 pu representing the field current corresponding to rated MVA output and power factor [1].

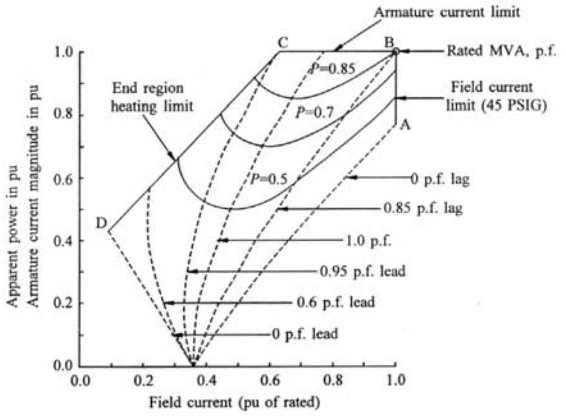

Figure 9 – *The V curves and compounding curves for a generator at rated armature voltage*

AN ELEMENTARY VIEW OF TRANSIENT STABILITY

Transient stability is the ability of the power system to maintain synchronism (angle stability) when subjected to a severe disturbance such as a fault on transmission facilities, loss of generation, or loss of a large load. The system response to such disturbances involves large excursions of generator rotor angles, power flows, bus voltages, and other system variables. Stability is influenced by the nonlinear characteristics of the power system. If the resulting angular separation between the machines in the system remains within certain bounds, the system maintain synchronism. Loss of synchronism because of transient instability, if it occurs, will usually be evident within 2 to 3 seconds of the initial disturbance.

Consider the system shown in Figure 10, consisting of a generator delivering power to a large system represented by an infinite bus through two transmission circuits. An infinite bus represents a voltage source of constant voltage magnitude and constant frequency.

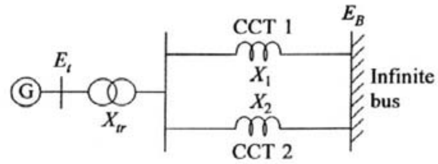

Figure 10 – *Single-machine infinite bus system*

We will present fundamental concepts and principles of transient stability by analyzing the system response to large disturbances, using very simple models, given by some of previous simplifications. All resistances are neglected. The generator is represented by the classical model and the speed governor effects are neglected. The corresponding system representation is shown in Figure 11(a). The voltage behind the transient reactanse ($X_d^{'}$) is denoted by $E^{'}$. The rotor angle δ represents the angle by which $E^{'}$ leads E_B. When the system is perturbed, the magnitude of $E^{'}$ remains constant at its predisturbance value and δ changes as the generator rotor speed deviates from synchronous speed ω_0.

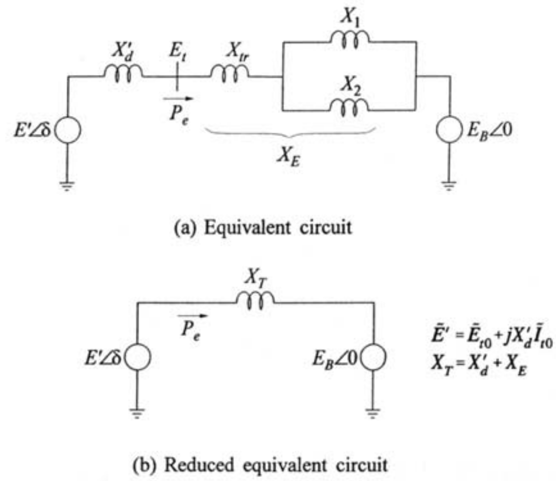

Figure 11 – *System representation with generator represented by classical model*

The system model can be reduced to the form shown in Figure 11(b). It can be analyzed by using simple analytical methods and is helpful in acquiring a basic understanding of the transient stability phenomenon. The generator`s electrical power output is

$$P_e = \frac{E^{'} E_B}{X_T} \sin\delta = P_{\max} \sin\delta \qquad (38)$$

where

$$P_{\max} = \frac{E^{'} E_B}{X_T} \qquad (39)$$

Since we have neglected the stator resistance, P_e represents the air-gap power as well as the terminal power. The power-angle relationship with both transmission circuits in service (I/S) is shown graphically in Figure 12 as curve 1. With a mechanical power input of P_m, the steady-state electrical power output P_e is equal to P_m, and the operating condition is represented by point *a* on the curve. The corresponding rotor angle is δ_a.

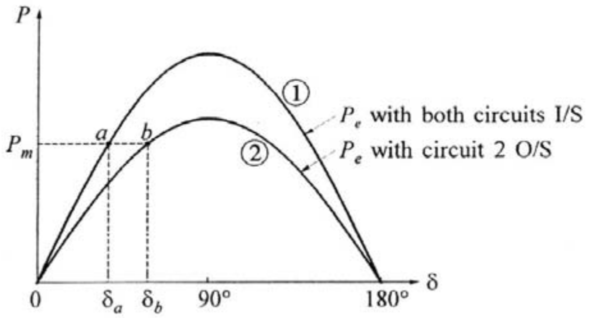

Figure 12 – *Power-angle relationship*

If one of the circuits is out of service (O/S), the effective reactance X_T is higher. The power-angle relationship with circuit 2 out of service is shown in Figure 12 as curve 2. The maximum power is now lower. With a mechanical power input of P_m, the rotor angle is now δ_b corresponding to the operating point *b* on curve 2; with a higher reactance, the rotor angle is higher in order to transmit the same steady-state power.

During a disturbance, the oscillation of δ is superimposed on the synchronous speed ω_0, but the speed deviation ($\Delta\omega_r = d\delta/dt$) is very much smaller than ω_0. Therefore, the generator speed is practically equal to ω_0, and the per unit (pu) air-gap torque may be considered to be equal to the pu air-gap power. We will therefore use torque and power interchangeably when referring to the swing equation.

The *equation of motion* or the *swing equation* may be written as

$$\frac{2H}{\omega_0} \frac{d^2\delta}{dt^2} = P_m - P_{\max} \sin\delta \qquad (40)$$

where
P_m = mechanical power input, in pu
P_{max} = maximum electrical power output, in pu
H = inertia constant, in MWs/MVA
δ = rotor angle, in elec. rad
t = time, in s

CONCLUSION

With these simplifications, as it has been shown in the view of transient stability, it is possible to more easily understand a problem of power system stability. Generaly, these simplifications and neglections contribute to gather with these problems more easily, minimizing also computational effort.

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He received many prestigious world-known awards and certificates for his creative work. Hence, he is included in the work of world groups of creativity, research and new technology in Brussels, Moscow, Pittsburgh and other world cities. He is also involved in research projects from the field of theoretical electrical engineering in Germany, Belgium and Russia.