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PLANE-WAVE DIFFRACTION FROM AN ASYMMETRICAL DOUBLE STRIP GRATING

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Abstract: The paper treats the problem of plane-wave diffraction from an asymmetrical double-strip grating. The problem is solved by using an equivalent network model. The theory is illustrated by numerical results for the transmitted basic mode power versus the relative grating period in the case of perpendicular incidence and TE polarization.

Keywords: Diffraction, grating, network model

INTRODUCTION

The problem of diffraction from different metal gratings has attracted attention for a prolonged period of time. One of the most powerful tools for treating this problem is the Riemann-Hilbert method [1], but its mathematical basis is rather complex. In [2], equivalent network models are proposed for plane-wave diffraction from a single-strip grating (one strip per period). Consequently, some relatively complex formulas are obtained for the elements of these network models [3]. These formulas require numerical evaluation of some double integrals. Very simple explicit formulas for the equivalent network elements related to plane-wave diffraction from a single-strip grating are obtained in [4], [5] and [6]. Plane-wave diffraction from a symmetrical double-strip grating (two strips per period) with equal strips or equal gaps is analyzed in [7] and [8], where explicit simple formulas for the equivalent network elements are also obtained. In this paper we consider plane-wave diffraction from an asymmetrical double-strip grating whose strips (and gaps) have different widths. This case is more complex than the previous ones, but it turns out that it is also possible to find explicit, relatively simple formulas for the equivalent network elements. The key moment here was solving in the closed form a certain integral equation with a logarithmic kernel.

EQUIVALENT NETWORK MODEL

Geometry of an asymmetrical double-strip grating is shown in Fig.1 A plane-wave, which can be either TE or TM polarized, is incident at an angle θ. Distance Δ, which reflects the grating asymmetry, is measured from the middle of distance d (origin O_1) to the middle of distance d_1 (gap middle). For the grating shown in Fig.1, distance Δ is negative. It is sufficient to consider only the TE case, since a duality theorem [9] can be used to treat the TM case. This theorem asserts that TM diffraction from an original grating is equivalent to TE diffraction from the complementary grating (strips and gaps are interchanged), and transmitted power in one case corresponds to reflected power in the other case, and vice versa.

An equivalent network model is shown in Fig.2.

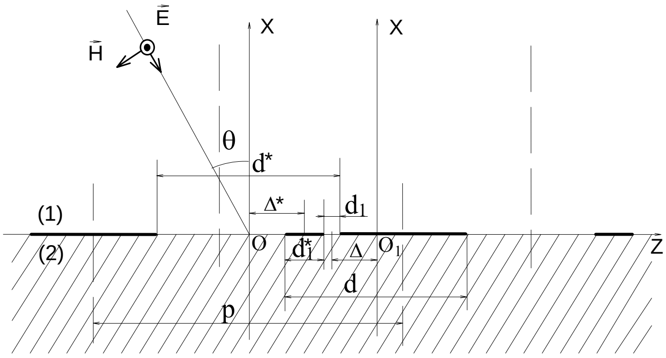

Fig.1 - *Geometry of the problem*

The transmission lines labeled by n =0, ±1, ±2, ±3... correspond to different harmonics (modes). Quantities G_n are given by

$$G_n = -\frac{j\omega\mu_0 p}{4\pi n}$$

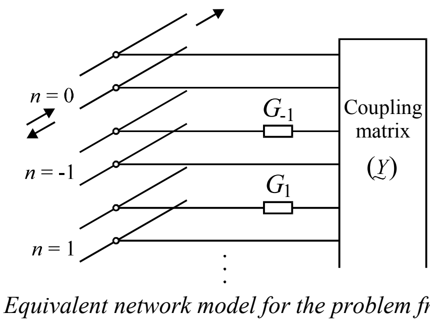

Fig.2 - *Equivalent network model for the problem from Fig.1*

and the elements of the coupling matrix (which is an admittance matrix) are to be found from

$$Y_{mn} = \frac{1}{2B} \int_{-A}^{A} F_n(\xi) e^{jm\xi} d\xi; \quad m, n = 0, \pm 1, \pm 2, ... \qquad (1)$$

where $F_n(\xi)$ is a solution of the integral equation

$$\int_{-A}^{A} F_n(\xi') \ln\left(2\sin\frac{|\xi - \xi'|}{2}\right) d\xi' = -e^{-jn\xi}; \quad n = 0, \pm 1, \pm 2, ... \qquad (2)$$

The constant B in (1) depends on frequency and is given by

$$B = \frac{j\omega\mu_0 p}{4\pi}$$

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and the constant A in (1) and (2) is

$$A = \frac{\pi d}{p}.$$

The sign "⁄" on the integrals in (1) and (2) means that the interval (δ-a, δ+a) (corresponding to the gap of width d_1) is removed from the basic interval (-A, A), i.e. the integration is performed on two separate intervals (-A, δ-a) and (δ+a, A). The quantity a is given by

$$a = \frac{\pi d_1}{p}$$

and

$$\delta = \frac{2\pi\Delta}{p}.$$

SOLUTION OF THE INTEGRAL EQUATION

Integral equation (2) can be solved in a closed form. The method of solution consists in transforming this equation into a Cauchy-type singular integral equation in the complex domain. Referring to [10] for details we give here the final solution of eqn. (2), valid for n≥0

$$F_n(\xi) = \frac{\operatorname{sgn}(\xi-\delta)}{2\pi\sqrt{(\cos\xi - u)(v - \cos(\xi-\delta))}}\left[C_n \sin\left(\xi - \frac{\delta}{2}\right) + \right. \qquad (3)$$
$$\left. + D_n + jne^{-j\frac{\delta}{2}}\sum_{p=0}^{n}\hat{\rho}_{n-p}e^{-j(p+1)\xi}\right]$$

where $u=\cos A$, $v=\cos a$. For negative values of n, the solution is obtained from (3) by changing the sign of n, and by making a complex conjugation. The quantities $\hat{\rho}_k(u,v,\delta)$ are some polynomials defined as the coefficients in a power series expansion of the function $\sqrt{(z^2-2uz+1)(z^2-2vze^{j\delta}+e^{2j\delta})}$ in the disc $|z|<1$:

$$\sqrt{(z^2-2uz+1)(z^2-2vze^{j\delta}+e^{2j\delta})} = \sum_{n=0}^{\infty}\hat{\rho}_n(u,v,\delta)z^n, \; |z|<1$$

Solution (3) contains two unknown constants C_n and D_n. They can be evaluated numerically as follows. By substituting (3) into (2) we obtain an identity in ξ for all $\xi \in (-A, \delta - a) \cup (\delta + a, A)$. Next, we choose two arbitrary points ξ_1 and ξ_2 from the intervals (-A, δ-a) and (δ+a, A), and perform numerical integrations, which results in two equations for C_n and D_n.

Having found a solution of eqn. (2), given by (3), we can find the coupling matrix elements from (1). We give the final result

$$Y_{mn} = \frac{1}{2B}\left[\frac{C_n}{2}\left(e^{j\delta}\overline{Q}^*_m - \overline{Q}^*_{m-2}\right) + jD_n e^{-j\frac{\delta}{2}}\overline{Q}^*_{m-1} + n\sum_{p=0}^{n}\hat{\rho}_{n-p}\overline{Q}^*_{-m+p}\right], \; n \ge 0$$

$$Y_{mn} = \frac{1}{2B}\left[\frac{C^*_{-n}}{2}\left(e^{j\delta}\overline{Q}^*_m - \overline{Q}^*_{m-2}\right) + jD^*_{-n} e^{-j\frac{\delta}{2}}\overline{Q}^*_{m-1} - n\sum_{p=0}^{n}\hat{\rho}_{-n-p}\overline{Q}^*_{m+p}\right], \; n < 0$$

where Q_n are some polynomials in u and v defined by

$$\frac{1}{\sqrt{(z^2-2uz+1)(z^2-2vze^{j\delta}+e^{2j\delta})}} = \sum_{k=0}^{\infty}Q_n(u,v,\delta)z^k, \; |z|<1$$

NUMERICAL RESULTS

Fig.3 shows the relative transmitted power of the basic (n=0) mode versus the relative period for the case of perpendicular incidence and TE polarization. The strip widths are fixed and the position of the smaller strip is varied. Small circles exhibit the values from (1), obtained by the Rimann-Hilbert method.

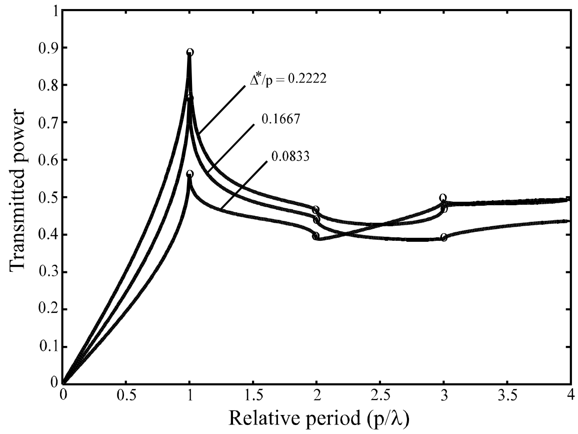

Fig.3 - *The relative transmitted power of the basic mode versus the relative period p/λ for the TE polarization. Parameters from Fig.1 are* $d^*/p = 0.5$, $d_1^*/p = 0.02776$ *and* $\theta = 0°$. *Parameter* Δ^*/p *is varied.*

CONCLUSION

This paper treats the plane-wave diffraction from an asymmetrical double-strip grating. The problem was solved by using an equivalent network model. The coupling matrix which is the key element of the network model is found after an integral equation with a logarithmic kernel was solved in a closed form. Numerical results are given for the transmitted power of the basic mode versus the relative period, and are compared with the corresponding results obtained by the Rimann-Hilbert method.

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