

# COMPARISON OF DIFFERENT FORMULATIONS AND TECHNIQUES FOR 3D STATIC FORCE COMPUTATION OF A T-SHAPED ELECTROMAGNET

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**Abstract:** In the paper, a comparison of three formulations using 3D finite element method - with magnetic vector potential with nodal and edge elements and magnetic scalar potential and two techniques – virtual work and Maxwell stress tensor - for the static force computation of a T-shaped electromagnet is presented. Comparison with experimental results is also made and error estimation is given. The influence of the number of elements is also presented.

**Keywords:** Electromagnetic forces, 3D FEM, Electromagnets, Magnetic vector potential, Magnetic scalar potential, nodal formulation, edge formulation.

## INTRODUCTION

Three-dimensional electromagnetic force computation is subject of permanent interest for researchers in recent years [1-9].

In the paper, different approaches for static force computation of T-shape electromagnet, implemented in ANSYS<sup>®</sup> program [10], are comparatively presented. Experimental values of the static electromagnetic force, for a range of air gaps and different magnetomotive forces (m.m.f.) are also obtained and used as a reference for the error estimation of the computed values.

## NUMERICAL COMPUTATION APPROACHES

The governing equations are the reduced set of Maxwell's equations:

$$\nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{H} = \mathbf{J} \quad (1)$$

where  $\mathbf{H}$  is magnetic field intensity,

$\mathbf{B}$  is magnetic flux density

$\mathbf{J}$  is the source current density.

The above field equations are supplemented by the constitutive relation that describes the behavior of magnetic materials respect to the applied field. A general relationship that includes permanent magnets and ferromagnetic materials is given by:

$$\mathbf{B} = \mu(\mathbf{H} - \mathbf{H}_c) \quad (2)$$

where  $\mu$  is magnetic permeability, in general a tensor and a nonlinear function of  $\mathbf{H}$  (for an isotropic materials,  $\mu$

simplifies to a scalar nonlinear function of  $H = |\mathbf{H}|$ ).  $\mathbf{H}_c$  is the coercive field for the material.

The solutions of equations (1)-(2) is commonly obtained using potential functions. Two kind of potential functions, the magnetic vector potential (MVP) and the magnetic scalar potential (MSP) are used.

From (1), the magnetic flux intensity can be expressed in terms of vector potential ( $\mathbf{A}$ ) as follows:

$$\mathbf{B} = \nabla \times \mathbf{A} . \quad (3)$$

The field equation in term of  $\mathbf{A}$  for magnetostatic problems is obtained from (1)-(3) and is given by:

$$\nabla \times \frac{1}{\mu} \nabla \times \mathbf{A} + \nabla \times \mathbf{H}_c = \mathbf{J} \quad (4)$$

The resultant magnetic field intensity may be regarded as the sum of field  $\mathbf{H}_s$ , coming from the conduction current sources  $\mathbf{J}$  or from the externally applied boundary conditions and the field  $\mathbf{H}_m$ , coming from the induced magnetism of ferromagnetic materials:

$$\mathbf{H} = \mathbf{H}_s + \mathbf{H}_m \quad (5)$$

Since there are not conduction currents within ferromagnetic materials, in this case  $\nabla \times \mathbf{H}_m = \mathbf{0}$  and it follows that:

$$\mathbf{H}_m = -\nabla \phi, \quad (6)$$

where  $\phi$  is the magnetic scalar potential (MSP). Introducing (5), (6) in (3), it gives the governing equation for  $\phi$ :

$$\nabla \cdot (\mu \nabla \phi) = \nabla \cdot (\mu \mathbf{H}_s) + \nabla \cdot (\mu \mathbf{H}_c) \quad (7)$$

For analysis of the 3D static magnetic field of a T-shape electromagnet, three formulations implemented in ANSYS<sup>®</sup> program have been employed – MVP (nodal and edge based) and MSP (nodal based).

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The MVP nodal formulation has three degrees of freedom per node:  $A_x$ ,  $A_y$ , and  $A_z$ , the magnetic vector potentials in the X, Y and Z directions. The current sources (current conducting regions) are considered as an integral part of the finite element model. The MSP formulation allows to simply model current sources as primitives rather than elements and so, these do not need to be part of the finite element mesh. The MVP edge formulation associates degrees of freedom with element edges rather than element nodes. It is often considered as better than the nodal based MVP nodal formulation in the cases of presence of media of different properties.

The static electromagnetic force is computed using two techniques: the Maxwell stress tensor and virtual work principle.

With the first, the force is computed by performing integration of the Maxwell stress tensor  $\mathbf{T}$  over a surface in the air around the plunger. If  $\mathbf{n}$  denotes the unit outward normal to the enclosing surface  $S$ , the force is obtained by:

$$\mathbf{F} = \oint_{(S)} \mathbf{T} \cdot \mathbf{n} d\mathbf{S} \quad (8)$$

The components of the Maxwell stress tensor are defined using the flux density  $\mathbf{B}$  and its components along the three co-ordinate axes:

$$\{T\} = \frac{1}{\mu_0} \begin{pmatrix} B_x^2 - \frac{1}{2}B^2 & B_x B_y & B_x B_z \\ B_y B_x & B_y^2 - \frac{1}{2}B^2 & B_y B_z \\ B_z B_x & B_z B_y & B_z^2 - \frac{1}{2}B^2 \end{pmatrix} \quad (9)$$

where  $\mu_0$  is the permeability of free space.

With the virtual work principle, the electromagnetic forces are obtained as the derivative of the energy with respect to the displacement of the movable part. This calculation for MSP formulation is applied to a layer of air elements surrounding the movable part. To determine the total force acting on the body, the forces in the air layer surrounding it can be summed.

The basic equation for force of an air material element in the  $s$  direction is:

$$F_s = \int_v \{B\}^T \left\{ \frac{\partial H}{\partial s} \right\} dv + \int_v \left( \int \{B\}^T \{dH\} \right) \frac{\partial}{\partial s} dv \quad (10)$$

where:  $F_s$  is force in element in the  $s$  direction

$s$  is virtual displacement of the nodal coordinates taken alternately to be in the  $x$ ,  $y$ ,  $z$  global directions;

$\left\{ \frac{\partial H}{\partial s} \right\}$  - derivative of field intensity with respect to

displacement;

$v$  - volume of the element.

For the MVP edge element formulation, the electromagnetic force is calculated on a selected set of nodes.

## MODEL DESCRIPTION

For FEM and experimental verification, the model shown in Figure 1, with the dimensions given in the Table I, was used [9]. The minimum value of the air gap is  $\delta_{\text{MIN}} = 0.2$  mm, value imposed by an insulated material witch covers the bottom interior surface core. The magnetic core is made of laminated steel and the magnetisation curve B-H presented in Figure 2 was taken in account. The coil is excited by a DC current and its number of turns is  $w = 11500$ , having the electric resistance  $R_B = 2300$  Ohm. The m.m.f. were chosen to be 345, 402.5, 460, 517.5 and 575 A (DC) in order to investigate the static force characteristics.

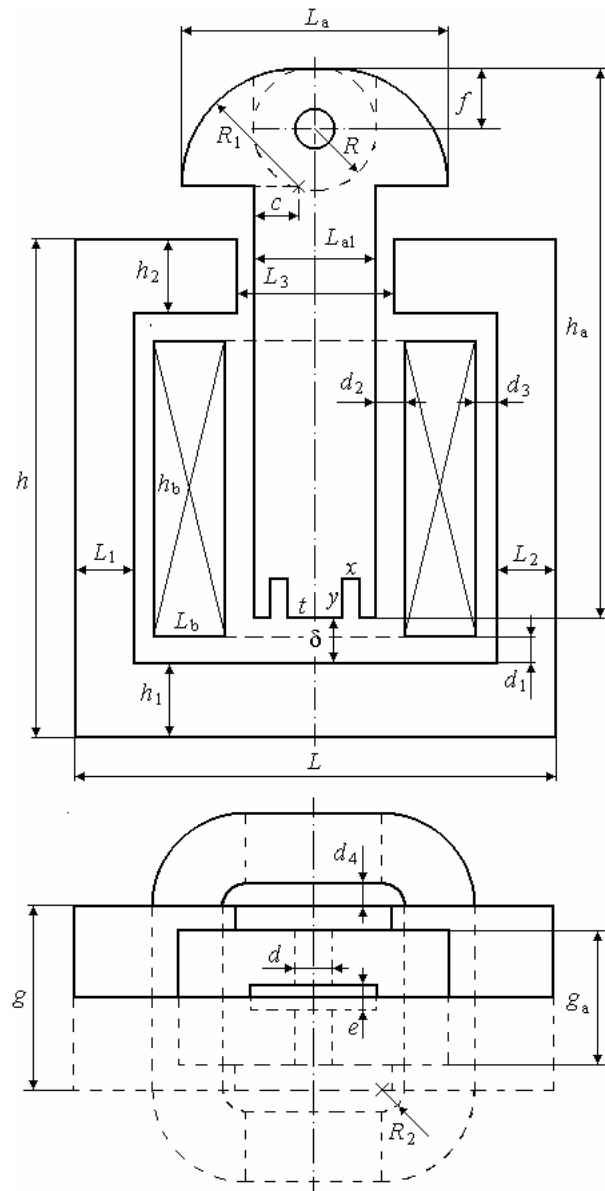
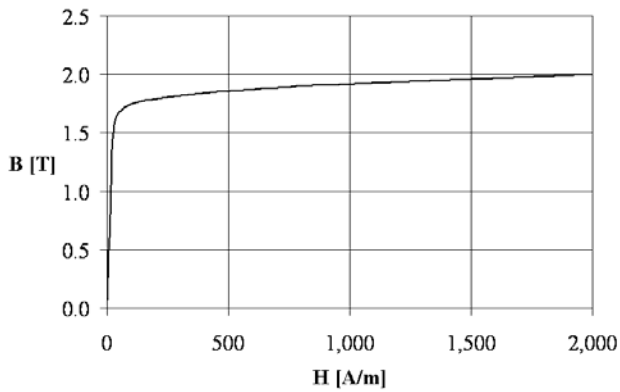


Fig.1 – T-Shaped electromagnet

**Table I**  
Dimensions of the electromagnet [mm]

$h$	52.5	$g$	19.8	$f$	6.30	$h_b$	31.2
$h_1$	7.90	$h_a$	57.8	$R$	6.50	$L_b$	7.50
$h_2$	7.90	$L_a$	28.3	$R_1$	12.3	$d_1$	2.40
$L$	50.9	$L_{a1}$	13.0	$g_a$	14.3	$d_2$	3.00
$L_1$	6.35	$c$	4.65	$x$	1.60	$d_3$	2.10
$L_2$	6.35	$d$	4.00	$y$	4.20	$d_4$	2.25
$L_3$	16.5	$e$	2.60	$t$	6.00	$R_2$	2.40



**Fig.2 - B-H curve**

## FEM AND EXPERIMENTAL VERIFICATION

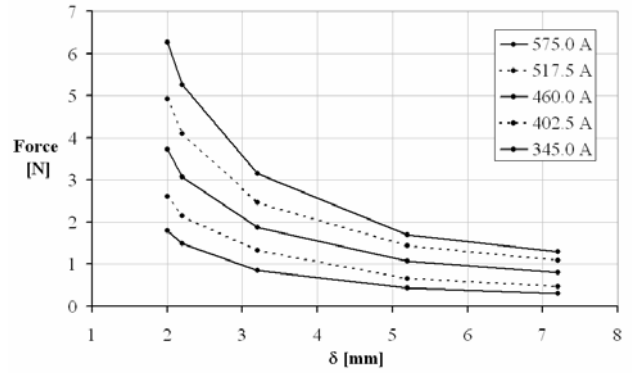
The electromagnetic force was measured using tensosensor for a range of the air gap ( $\delta$ ) between 2 and 7.2 mm at the specified values of m.m.f. The experimental results for the force-stroke characteristics are illustrated in Figure 3.

The numerical results of the 3-D static analysis have been obtained using ANSYS program, for each of the three approaches: MVP, MSP and Edge Element formulation. For automation of the numerical computation, command files have been created using APDL (ANSYS Parameter Design Language). This allows multiple runs to be executed easy and changing any of the parameters is carried out only by changing a line in the command file.

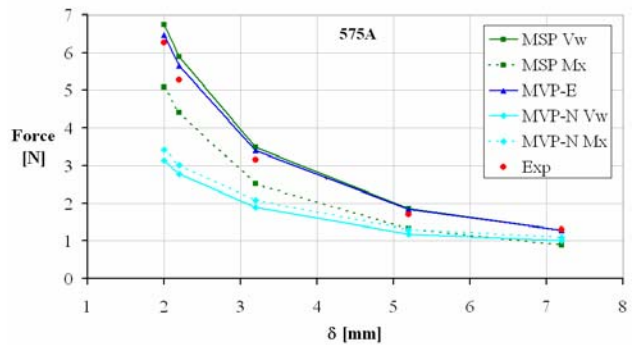
The mesh was realized using tetrahedral elements. A quarter a model was analysed, a four-time reduction of the domain being obtained.

The number of elements vary in range 120 000 – 200 000, 140 000 – 190 000 and 130 000 – 170 000 in MVP (nodal), MSP, and MVP (edge) computation, respectively, limited by hardware resources.

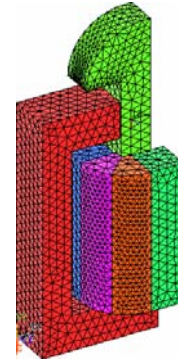
For m.m.f. value of 575 A, the numerical results for the force-stroke characteristics with MVP nodal formulation, the earlier results with MSP and MVP edge formulations [9] and experimental results are comparatively shown in Figure 4.



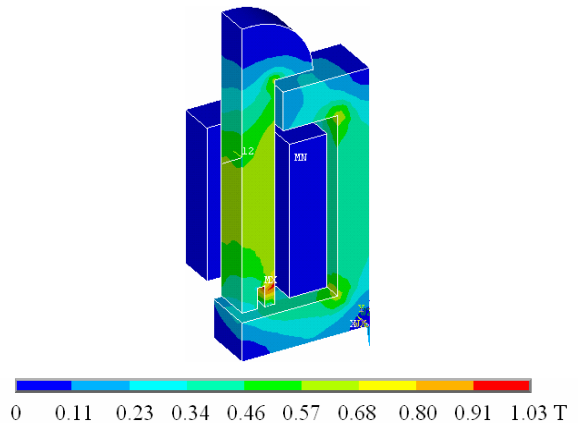
**Fig.3 - Experimental force-stroke characteristics**



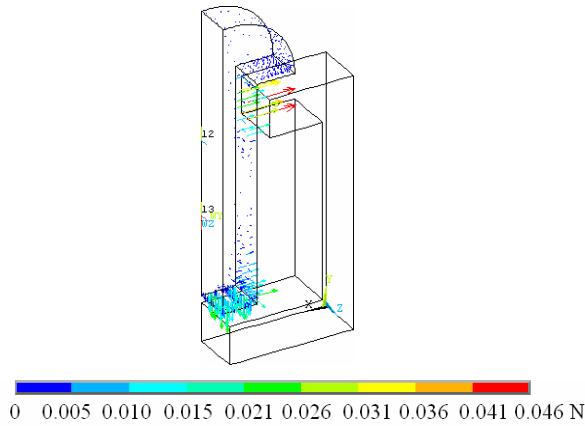
**Fig.4 – Force-stroke characteristics for m.m.f. 575 A**



**Fig.5 - Front-view mesh model building (2 mm, 575 A, MVP nodal formulation)**



**Fig.6 - Magnetic flux density distribution (2 mm, 575 A, MVP nodal formulation)**



**Fig.7 - Static electromagnetic force vectors representation (2 mm, 575 A, MVP-N Formulation)**

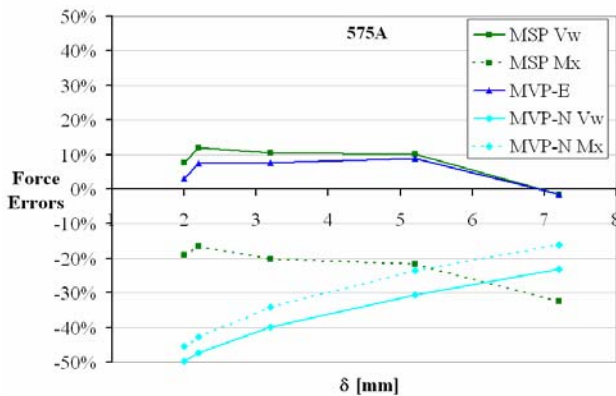
Using the visualization facilities offered by ANSYS program, the Figure 5 show the 3D symmetry plane perspective of the model with associated mesh.

The magnetic flux density distribution on symmetry plane is shown in Figure 6 and the electromagnetic force vectors are plotted in Figure 7.

## DISCUSSION

The percentage errors of the three numerical solutions related to measurements have been calculated.

As Figure 8 shows, for m.m.f. of 575 A, the most precise solution is given by virtual work technique of MVP edge element formulation with an average relative error of 5.1 % and MSP formulation with 7.7 % and Maxwell stress tensor technique of MSP formulation with an average relative error of 21.9 %. The MVP nodal formulation presents great relatives errors for the both technique, virtual work and Maxwell stress tensor, of about 38.2% and 32.4%, respectively.

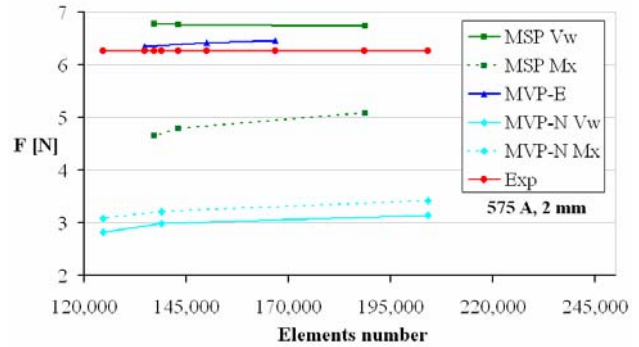


**Fig.8 - Percentage errors comparison**

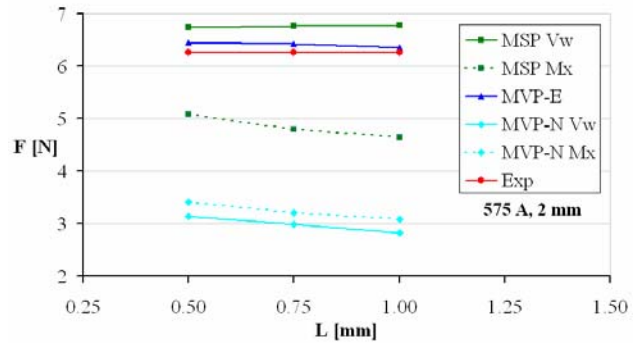
A study of the influence of the number of elements and element size near the air gaps on the solution is carried out. A thin mesh was built in the air gaps and in the layer of air around the plunger, where the force is computed.

In Figure 9 and Figure 10, the influence of the number of elements and of the maximal linear dimension of the elements in the air gap zone, can be seen, for value of the m.m.f. 575 A and air gap 2 mm. The computation was

carried out on a PC with 512 MB RAM. The MVP edge and MSP formulation with virtual work technique are more stable in terms of the element number variation, compared with MSP with Maxwell stress tensor and MVP formulation, which are sensibly influenced. The same conclusion can be drawn for the element size influence. Greater memory level can increase the accuracy of the MVP nodal formulation.



**Fig.9 - Solution convergence – number of elements influence**



**Fig.10 - Solution convergence - maximal element size influence**

## CONCLUSIONS

Three formulations and five numerical solutions of static force characteristics of a T-shaped electromagnet are compared: Magnetic Vector Potential nodal formulation with both techniques (virtual work and Maxwell stress tensor), Magnetic Scalar Potential formulation with the same techniques, and Magnetic Vector Potential edge formulation.

Their performance was analysed using the 3D Finite Element Method and ANSYS® program. The different approaches have different behavior with variation of the air gap.

For estimation of the accuracy of the different approaches, experiment is carried out and the experimental results are taken as reference.

The comparison with the experimental data leads to the conclusion that MVP edge formulation and MSP formulation with virtual work give most accurate results. The MSP formulation with Maxwell stress tensor, as well as the two techniques for the MVP nodal formulation do not give satisfactory accuracy.

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