Invited paper

COMPUTATIONAL ELECTROMAGNETICS - REVIEW AND TRENDS

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COMPUTATIONAL ELECTROMAGNETICS

Electromagnetic theory was fully formulated by James Clerk Maxwell in 1864 in terms of the Maxwell's equations. Even though it has been around for over a hundred years, scientists and engineers are continuously pursuing new methods to solve these equations. The reason is that Maxwell's equations govern the law for the manipulation of electricity. Hence, many branches of electrical engineering are directly or indirectly related to the electromagnetic theory. Scientists and engineers solve these equations in order to gain a better understanding of and physical insight into systems related to the use of electromagnetic fields and waves. The solutions of Maxwell's equations can also be used to predict design and experimental outcomes.

Electromagnetics has persisted as a vibrant field despite it being over a hundred year old is because many electrical engineering technologies depend on it. To name a few, these are: physics based signal processing and imaging, computer chip design and circuits, lasers and optoelectronics, MEMS (micro-electromechanical sensors) and microwave engineering, remote sensing and subsurface sensing and NDE (non-destructive evaluation), EMC/EMI (electromagnetic compatibility/electromagnetic interference) analysis, antenna analysis and design, RCS (radar cross section) analysis and design, ATR (automatic target recognition) and stealth technology, wireless communication and propagation, and biomedical engineering and biotechnology.

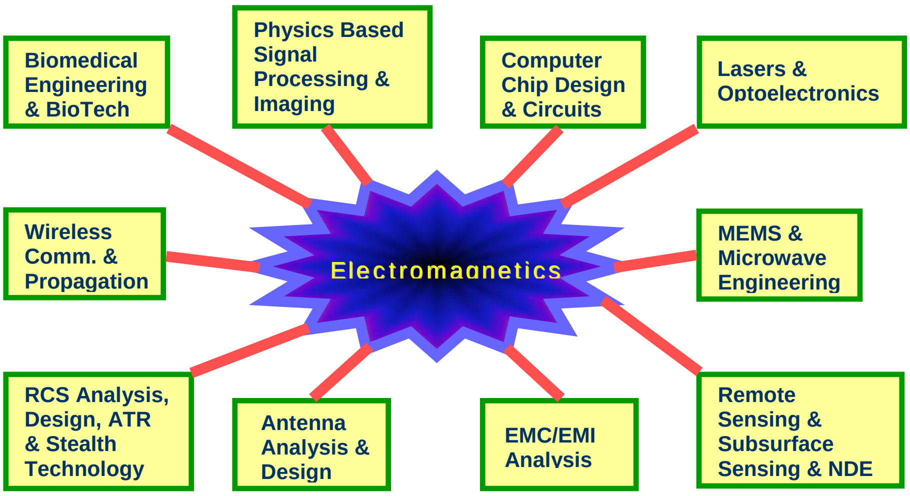

Fig. 1: The impact of electromagnetics is far-reaching and affects many different branches of electrical engineering technologies.

For instance, the field of computer chip design has long relied on the use of circuit theory, which is a subset or an approximation of electromagnetic theory when the frequency is very low. As the clock frequency of a computer becomes higher, circuit theory becomes inadequate in describing many of the physical phenomena that occurs within a computer chip. Electromagnetic theory has to be used to correctly describe the physics within a computer chip. An emerging electromagnetic analysis method is computational electromagnetics where the computer is used intensively to analyze electromagnetic problems. The growth of this field has been spurred by the rapid growth of computer speed, and now the further growth and design of faster computers will rely on computational electromagnetics — a symbiotic existence indeed.
Computational Electromagnetics (CEM) is the science of numerically solving a complex set of Maxwell's equations using limited computer resources. These solutions describe the physical interactions and phenomena between charged particles and materials. A fundamental understanding of those phenomena is critical in the design of many devices such as radars, computer chips, optical fibre systems, mobile phone systems, etc.
CEM comprises the methodologies for numerically solving Maxwell's equations in the time and frequency domains. The range of applications is diverse and encompasses areas such as: modelling radiation and scattering from platforms, RCS prediction, EMC, antenna design and analysis, the design of high-speed electronic circuits, communications technology and remote sensing, to list just a few. Various solution methodologies have been developed (e.g. finite element, finite difference-time domain, etc.) to treat certain classes of problems. It is a multi-disciplinary subject, combining the mathematics of vector calculus and numerical analysis with the physics of wave generation and propagation together with an appreciation of computer science and information technology.

HISTORY OF NUMERICAL METHODS

Electromagnetics, the study of the solution methods to solving Maxwell's equations, and the application of such solutions for understanding and engendering new technologies, has a long history of over a hundred years. But the analysis method with Maxwell's equation is constantly evolving over the years. In the beginning, there was the age of simple shapes: during this period, roughly between the late 19th century to 1950s, solution methods, such as the separation of variables, harmonic analysis, and Fourier transform methods were developed to solve for the scattering solution from simple shapes. We can identify the names of Sommerfeld, Rayleigh, Mie, Debye, Chu, Stratton, Marcuvitz, and Wait for contributions during this era.

Despite the successful closed form solutions for simple geometries, the solutions available were insufficient to analyze many electromagnetic systems. Hence, scientists and engineers started to seek approximation solutions to Maxwell's equations. This was the age of approximations, roughly between 1950s and 1970s. During this period, asymptotic and perturbation methods were developed to solve Maxwell's equations. The class of solvable problems for which approximate solutions exist, was greatly enlarged.

However, the limited range of approximate solutions of Maxwell's theory still could not meet the demand of many engineering and system designs. As soon as the computer was developed, numerical methods were studied to solve Maxwell's equations. This was the age of numerical methods (1960s+). Method of Moments (MoM), finite difference time domain method (FDTD), and finite element method (FEM) were developed to solve problems alongside with many other numerical methods. In particular, Harrington was noted for popularizing MoM among the electromagnetics community, while it is know as the boundary element method (BEM) in other communities. Yee developed FDTD, for solving Maxwell's equation. Finite element has been with the structure and mechanics community, and Silvester was an early worker who brought its use into the electromagnetics community. Other names commonly cited in this field are: Wilton, Mittra, and Taflove.

There has always been marriage between electromagnetics and mathematics from the very beginning. Actually, quite sophisticated mathematical techniques were used to analyze electromagnetic problems because electromagnetic theory was predated by the theory of fluid and theory of sound. These fields were richly entwined with mathematics with the work of famous mathematicians such as Euler, Lagrange, Stokes, and Gauss. Moreover, many of the mathematics of low Reynold number flow in fluid theory and scalar wave theory of sound can be transplanted with embellishment to solve electromagnetic problems.

Examples of problems solved during the age of simple shapes are the Mie and Debye scattering by a sphere and Rayleigh scattering by small particles. Rayleigh also solved the circular waveguide problem for electromagnetic waves because he was well versed in the mathematical theory of sound, having written three volumes on the subject while sailing down the Nile River. Sommerfeld solved the half plane problem as far back as 1896 because the advanced mathematical techniques were available then. He also solved the Sommerfeld half space problem in 1949 in order to understand the propagation of radio waves over the lossy half-earth. The problem was solved in terms of, what is now known as, the Sommerfeld integrals, an example of which is as follows:

$$\psi(\mathbf{r}) = \int_0^\infty dk_\rho J_n(k_\rho \rho)\left[e^{ik_z|z-z'|} + R(k_\rho)e^{ik_z z}\right]$$

$$k_z = \left(k^2 - k_\rho^2\right)^{1/2}$$

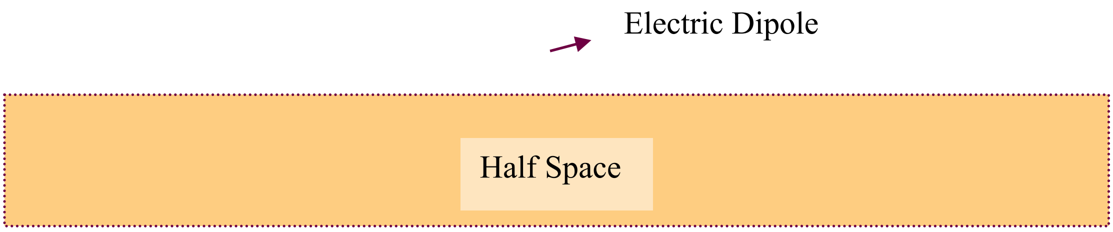

Fig. 2: A dipole over a half space. The problem was first solved by Sommerfeld to understand the propagation of radio waves over the lossy earth.

Evaluating the Sommerfeld integrals was an impossibility during his time, but it is a piece of cake now in the modern era. Subsequently, approximation techniques, such as the stationary phase method, the method of steepest descent, and the saddle point methods were used to derive approximations to the Sommerfeld integrals.

However, even though electromagnetics has been intimately entangled with mathematics, a student of electromagnetics has to be able to read the physics into the mathematical expressions that describe the solutions of Maxwell's equations. Approximate methods generally help to elucidate the physics of the wave interaction with complex geometry.

The physical insights offered by approximate solutions spurred the age of approximations, roughly between 1950s and 1970s. A large parameter such as frequency is used to derive asymptotic approximations. Moreover, heuristic ideas were used to derive the physical optics approximation, Kirchhoff approximation, and various geometrical optics approximations. These approximations eventually lead to the geometrical theory of diffraction and the uniform asymptotic theory of diffraction. The applications of these approximate methods to scattering by complex structures are usually ansatz based. The coefficients of the ansatz can be found from canonical solutions such as the Sommerfeld half plane problem, or scattering by a sphere or a cylinder, followed by the use of Watson transformation. The use of approximate solution enlarges the class of solvable problems, but the error is usually not controllable. Asymptotic series are semi-convergent series; hence there is not a systematic way to reduce the solution error by including more terms in the ansatz. Moreover, the range of application is limited because the frequency has to be sufficiently high before the ansatz forms a good approximation.

The advent of the transistorized computer in the 1960s almost immediately brought about the birth of numerical methods for electromagnetics. The method of moments (MoM) was popularized among the electromagnetics community by Harrington in the 1960s. The method is integral-equation-based, and is versatile for solving problems with arbitrary geometries. It entails a small number of unknowns since the unknown is the current, but unfortunately, the pertinent matrix equation is dense. The finite-difference time-domain method was proposed by Yee in the 1960s for solving Maxwell's equations in its partial differential form. The method is extremely simple, and gives rise to a sparse matrix system. Since the field is the unknown to be solved for, the drawback is that it entails a large number of unknowns. Moreover, the field is always propagated from point to point via a numerical grid, hence yielding grid dispersion error, which accentuates with increasing problem size.

In the finite difference method based on differential equations, the solution region is subdivided into rectangular mesh (a discretization, and thus, an approximation) and the values of a scalar potential field are sought at all grid points. Engineers, with intuition derived through experience in circuit theory, were able to use network analogous models and generalize the finite difference method to inhomogeneous problems with graded meshes in such a way as to be easily understood. Initially, solutions were obtained for small problems by hand relaxation, which in turn gave way to bigger problems by transferring to digital computers. The disadvantage of this method is that curved boundaries and varying sources cannot be modelled using simple, general data structures, so that general purpose software is difficult to write for complex problems. Moreover, open boundaries have to be modelled by a close boundary with artificial boundary conditions. Recent years have witnessed the development of very general finite difference schemes that overcome most of these deficiencies. These elegant variants, however, require complex programming efforts and special data structures.

There are other numerical methods which were developed from variational methods, based on the so-called Euler´s brachistochrone problem [Gould 1957]. This problem gives us that particular shape of a curve between two points in space, along which a mass may slide down without friction in the shortest time. A variational scheme may commonly be described as *differential* in that we replace the condition of the satisfaction of a differential equation governing an unknown function by the equivalent requirement that an integral function of the unknown function shall be at a minimum. Indeed, integral variational principles are also used where we use a functional that satisfies an integral relationship at its minimum.

Already in the early 1950s differential variational schemes have been used to analyze electromagnetic devices, e.g. waveguides. These schemes are remarkably similar to the technique of modern finite element analysis. They relied upon a Rayleigh-Ritz scheme, which assumed the solution to be a sum of coordinate functions, with their weights to be determined by variational calculus [van Bladel 1964]. These variational schemes differed from present-day finite elements only in that the solution domain was not split into subdomains over each of which a differential trial function may be assumed. As a consequence of their trial solution being applicable to the whole domain, the choice of coordinate functions was critical to the

accuracy of the solution. Thus, their methods are use only in a few problems for which a good estimate of the solution may be made.
With the development of the computer, statistical methods involving long solution times were also employed for the solution of field problems. The Monte Carlo simulation technique [Ehrlich 1959] is used to find the potential at one point within a device that has been divided into a mesh. We start at the point where we want the solution and "walk" through the nodes until we reach a boundary. The solution at the starting node is expressed as a statistical formula in terms of the charge densities at the nodes through which the walk is performed. Because this is a statistical formula, its accuracy increases with the number of walks we perform. Thus, for convergence of the solution several walks must be made. What we finally have is the solution at just one node. The validity of the method for general gradient (Neumann) boundary conditions has not been thoroughly investigated. It has also been found that the time taken to get the potential at one node is much greater than that for identifying the potential everywhere in the mesh by relaxation [Binns/Lawrenson/Trowbridge 1992]. As a result, the Monte Carlo method in electromagnetics has fallen into general disuse. However, the method is still extremely useful in other disciplines such as statistical chemistry where alternative techniques are hardly available.
Real numerical modelling of the continuum may by broadly divided into three groups:

- Differential methods
- Integral methods
- Variational methods
- Asymptotic methods

Differential equation methods (FDM, FDTD, etc.) are based on direct discretization of the differential governing field equations. Their most appealing characteristic relates to their ease of implementation. But they are also dispersive and costly (e.g., typically 10 – 20 points per wave length). Moreover they are restricted in applicability to complex geometries, especially in high-order implementations.
Integral equation methods (BEM, VIEM, MoM, Fast Multipole Methods, Adaptive Integral Methods, FFT-based Methods, Charge Simulation Methods, etc.) are based on discretizations of integral equation formulations of the governing equations. Similarly to variational approaches, they are versatile and flexible. They can also be made to be very efficient, particularly in applications involving piecewise homogeneous structures where they lead to a lower dimensional problem (posed on the interfaces separating different media). But they lead to full matrices and thus can only be made competitive through the use of mechanisms that accelerate the evaluation of fields. In addition, the singular character of the integrals imposes substantial challenges which typically result in low-order implementations, with consequently large computational costs.
Variational methods (FEM, Finite Volume Methods, etc.) are based on the variational formulation of Maxwell´s equations. They lead to algorithms that present several favourable properties, including great applicability and flexibility, a natural setting for adaptivity and parallelization, sparse matrices, etc. But these approaches typically require large (volumetric) computational domains and the use of approximate radiation conditions within a finite computational domain, which may lead to high computational costs and large memory requirements; in addition, low-order implementations (e.g. finite volume) suffer from significant dispersion and dissipation errors.
Asymptotic methods (ray-tracing, etc.) do not solve the full Maxwell model (in contrast with the methods above), but rather an approximation of it. Of particular interest from the point of view of applications (e.g. radar) are those that relate to the high-frequency (geometrical or physical optics) limit of Maxwell´s equations. Such methods are extremely efficient since, in

contrast with methods that solve the full Maxwell model, they do not involve the resolution of the fields in the scale of the wavelength of radiation. But, they are asymptotic in nature and therefore are not error-controllable; as a result, they can give rise to significant inaccuracies for finite (but large) frequencies.

As any review of the literature will reveal, schemes from each of these classes have been very successful at resolving a variety of problems. The same review will also discover that some methods may be better adapted to specific applications, and that no method can be considered "universally" superior. More importantly perhaps, one will also find that all available methods have very definite limitations in spite of continuous advances in the capabilities of computational algorithms and hardware. In fact, a number of applications continue to challenge every approach, and some lie well-beyond today´s capabilities (e.g. the rigorous prediction of scattering returns at very high frequencies).

And there is still another aspect which should be taken into account during evaluation of the numerical methods. Usually, it is not so clear how to classify the different methods following the classes given above. There are several modifications of some basic methodologies of numerical treatment of Maxwell´s equations where it is not possible to say precisely this is a differential method or that is a variational methods, etc. There are a number of rather new developments which have led to sometimes called "hybrid" methods, which not always means that there are combined two methods on a certain interconnection between two domains of solutions space, but they use combined discretization strategies in the same equations.

The variational methods are really based on the differential or integral form of the equation to be solved. Integral schemes for materially homogeneous problems have long been known and used. The equation for integration naturally follows from the potential due to a unit point charge being given at a certain distance (source point). Thus, any charge cluster of given density may be subdivided into small volumes so that the charge in that volume is effectively a point charge causing a potential in the considered field point. On the basis of the superposition principle, we may integrate such effects at any field point and get the potential there. When material inhomogeneities are involved, this scheme cannot be used. To overcome this in electrostatics, we may resort to dipole moment techniques whereby we eliminate an inhomogeneous region and replace the effects coming from inside the eliminated region by a surface charge. Because these charges are secondary sources, i.e. they only "appear" if primary sources generating the field are present, this method is called the method of secondary sources [Tozoni 1975]. In magnetics the analogous idea can be realized by introducing fictitious magnetic poles to replace the magnetized material.

In the late 1960s the finite element method was first applied to electromagnetics [Winslow 1967]. The finite element method is a general method for the solution of differential equations. Here, we subdivide the solution region into subdomains, called *elements*, and postulate a trial function ((with free parameters) over each of the elements. It is commonly found convenient to have interpolation nodes on the element and to define the trial function in terms of the unknown values of the unknown variable of the differential equation at the node. As a result, the nodal values become the free parameters. The finite element method essentially consists of finding the values of the free parameters with respect to some optimality criterion such as minimum error, energy extremum, functional orthogonality, etc. This method, now widely accepted as one of the most powerful numerical schemes available, is one that engineers may take true pride in, for having founded intuitively, leaving for later the rigorous justifications of the method, i.e. there was a time when engineers used the method on the grounds simply that it worked. Their mathematical verification was studied much later.

MATHEMATICAL MODELLING

Physics Knowledge

Electromagnetics is a branch of applied physics. However, due to the dependency of solution methods on mathematics, both knowledge of physics and mathematics are indispensable in the study of electromagnetics.

We should encourage our students to study modern physics; even though it is not directly relevant to electromagnetics, modern physics embodies the most beautiful of the physical theories that have been developed in this century. If a student can understand the thought processes and abstraction that go on in modern physics, she eventually will become a better thinker and a proficient problem solver. Our goal is to teach a student to think in graduate school. A proverbial saying is that "If you give a man a fish, it lasts him for a day, but if you teach a man how to fish, it lasts him a lifetime." Moreover, if we can stretch the mind of a student, it does not regain its original dimension.

The long history of electromagnetics has produced much classical knowledge that cannot be ignored by our students. They should have a good understanding of the fundamental solutions that accompany simple shapes. Furthermore, they should understand and should be able to elucidate the physics that arises from the approximate solutions, such as the physics of surface waves, creeping waves, lateral waves, evanescent waves (tunneling), guided modes, radiation modes, leaky modes, specular reflection, edge diffraction, etc.

Students of electromagnetics should be cognizant of the metamorphosis of the physics over different length scales or frequencies. When the wavelength is extremely long, one is in the regime of electrostatics and magnetostatics where the electric field and the magnetic field are decoupled or weakly coupled to each other. This is also the world in which circuit theory lives in. For shorter wavelengths, the coupling between the electric and magnetic field becomes stronger, and we have mid-frequency or high frequency electromagnetics, whose physics is quite different from low frequency electromagnetics. This is also when the wave nature of an electromagnetic field becomes important. Often, the vector nature of electromagnetic field plays an important role in this regime. As the frequency gets higher, then we are in the world of optics and ray optics. In this world, electromagnetic waves can be described by rays, and often be thought of as particles. Equations can be derived to govern only the propagation of the envelope of a pulse. At very high frequencies, the quantization of the energy associated with an electromagnetic field becomes important. A quantum of energy in electromagnetic field is $\mathrm{h}\omega$ where the Planck constant $\mathrm{h} = 1.05 \times 10^{-27}$ erg sec. Therefore, to properly understand the interaction of very high frequency electromagnetic field with material, we have to invoke quantum electrodynamics.

Mathematics Knowledge

Since there has often been an intimate intermarriage between electromagnetics and mathematics, a student of electromagnetics should understand the reason for the finesse, care, and precautions that mathematicians go through in their work. It is to lay a firm foundation for their mathematical work so that others can build upon them. They should understand the fundamental of harmonic analysis and complex variables, which traditionally have been used to analyze classical electromagnetic problems. The fundamental of perturbation and asymptotic methods should be understood in order to appreciate the wide body of knowledge generated by approximate calculations.

The advent of computational electromagnetics however, calls for the use of a different body of mathematical knowledge. A student who is well versed in computational electromagnetics should have an elementary understanding of functional analysis, Hilbert spaces, and operator theory. He should also understand partial differential equation (PDE) theory, the existence, uniqueness, and well-posedness of PDE solutions, and the integral equation theory that follows from PDE. A wonderful aspect of modern computational electromagnetics is the error controllability of the solution, i.e., the error can be made increasingly small by devoting increasing computational resources. Hence, the pervasiveness of computational method in the future will require that a student understands the elements of approximation theory and error bounds.

In many discussions with students it turn out that often times, a more elegant view of electromagnetic theory can be gotten from a topological viewpoint. Such is the viewpoint of electromagnetic theory from differential forms. Modern computational electromagnetics will inherently deal with complex geometry handling. A more profound understanding of electromagnetic theory through a topological viewpoint may even engender the development of new numerical methods.

Computer Science

Computers are used by all scientists and engineers as a tool. In order to harness the power of this tool effectively, it is important that a student of electromagnetics understands the basics of modern computer languages and computer architectures. A student should understand the basis of object oriented programming paradigm, and how its use will result in better computer code maintenance, reusability, and encapsulation. It is also important for students to learn the basis of parallel computing and large scale computing, and the need to use message passing for distributed processor computing. Many fast algorithms exist in computer science for sorting, searching, matrix manipulations that students should be aware of. They should also be aware of the element of computer architecture, the issue of distributed memory computing versus shared memory computing, the issue of memory latency, cache usage, cache hit and miss. Geometry handling will be indispensable from computational electromagnetics; hence, a student should understand certain aspects of computational geometry.

The solution of many practical electromagnetic field problems can only be undertaken by applying numerical methods. Before such a problem can be solved, it is important to establish a correct mathematical model for the problem considered. Maxwell´s equations and the associated boundary conditions provide the necessary basis for the modelling of practical electromagnetic problems. Further, the Green´s theorem, fundamental solutions, and equivalent sources are basic tools used in the numerical techniques.

According to Maxwell´s equations, all electromagnetic field problems can be expressed in partial differential equations which are subject to specific boundary conditions. By using Green´s function, the partial differential equations can be transformed into integral equations or differential-integral equations. The analytical solution of these equations can only be obtained in very simple cases. Therefore, numerical methods are significant for the solution of practical problems. In numerical solutions the following aspects have to be considered:

- A mathematical model expressed by differential equations, integral equations, or variational expressions is provided to describe physical states.
- A discretized model is suggested to approximate the solution domain, so that a set of algebraic equations is obtained.
- A computer program is designed to complete the computation.

In designing these steps one should consider:

- Does the mathematical model describe the physical state well?
- Does the approximate solution satisfy the desired accuracy?
- Does the method use the computer sources economically?

In order to obtain a good method for various engineering problems many methods have been developed. The purpose of various numerical methods used to obtain solutions for electromagnetic field problems is to transfer an operator equation (differential or integral equation) into a matrix equation. In solving field problems the problem can be described by differential or integral equations. Consequently, there are two different kinds of solution methods: using either differential equations or the integral equations. The former is known as the "field" approach, or the ***domain method***, and the second is known as a source distribution technique, or the ***boundary method***.
Hammond has interpreted these dual approaches in a historical perspective:

> ***"The history of electromagnetic investigation is the history of the interplay of two fundamentally different modes of thought.***
> ***The first of these, the method of electromagnetic fields which ascribes the action of a continuum, is associated with such thinkers as Gilbert, Faraday and Maxwell.***
> ***The second, the method of electromagnetic sources, concentrates the attention on the forces between electric and magnetic bodies and is associated with Franklin, Cavendish and Ampere. . . .***
> ***Field problems are conveniently handled by differential equations and sources by integral equations."*** [Hammond 1969]

Both of these two methods have advantages and drawbacks. No matter of which methods are applied, numerical solution methods consist of the following steps:

1. Express the unknown function u(**r**) contained in the operator equation by the summation of a set of linear independent functions with undetermined parameters of a complete set sequence, e.g.
$$u(\mathbf{r}) \;=\; \sum_{i=1}^{N} C_i \cdot \psi_i$$
where C_i are undetermined parameters and ψ_i are the terms of basis functions. The u(**r**) is called the trial function or approximate solution. If N goes to infinity, the approximate solution will tend towards the real solution.
2. Cast the continuous solution domain into a discrete form. The resulting set of discretized subdomains consists of a finite number of elements and nodes. In this fashion the unknown function with infinite degrees of freedom is replaced by an approximate function with finite degrees of freedom.
3. Choose a principle of error minimum in order to determine the unknown parameters contained in the trial function. This can be achieved by employing either variational principles or the principle of weighted residuals. After this step is executed, the operator equation is transformed to a matrix equation.
4. Obtain the approximate solution of a given problem by solving the linear or non-linear matrix equation derived from the previous step.

However, some of the recent advances in fast computational algorithms will remove the objections to the shortcomings of numerical methods.

NUMERICAL METHODS

In the literature can be found quite a lit of different numerical methods for calculation of fields (Fig. 3).

Fig. 3: The ´jungle´ of field computation methods

In the following chapters the most important numerical methods for field computation will be briefly described:

I. Finite Difference Method (FDM)
II. Finite Element Method (FEM)
III. Integral Equation Method (IEM)
IV. Finite-Difference Time-Domain Method (FDTD)

I. FINITE DIFFERENCE METHOD

The finite difference method (FDM) is an approximate method for solving partial differential equations where the differential quotients are substituted by difference quotients. It has been used to solve a wide range of problems. These include linear and non-linear, time-independent and dependent problems. This method can be applied to problems with different boundary shapes, different kinds of boundary conditions, and for a region containing a number of different materials. Even though the method was known by such workers as Gauss and Boltzmann, it was not widely used to solve engineering problems until the 1940s. The mathematical basis of the method was already known to Richardson in 1910 [Richardson 1910] and many mathematical books were published which discussed the finite difference method [Forsythe/Wasow 1960, Smith 1965]. Specific reference concerning the treatment of electric and magnetic field problems is made in [Binns et al. 1992]. The application of FDM is not difficult as it involves only simple arithmetic in the derivation of the discretization equations and writing the corresponding programs. Later FDM was the most important numerical method used to solve practical problems. With the development of high speed

computers having large scale storage capability many numerical solution techniques appeared for solving partial differential equations. However, due to the ease of application of the FDM it is still a valuable means for solving these problems. But because of the need generally to use regular grids the application of this method is limited.
Similar to other numerical methods, the aim of finite difference is to replace a continuous field problem with infinite degree of freedom by a discretized field with finite regular nodes. The partial derivatives of the unknown function are approximated by the difference quotients at a set of finite discretization points (nodes) using Taylor´s series, particular physical principles like the box integration technique, weighted residual approaches or variational principles [Zhou 1993, Binns 1992]. Thus, the original partial differential equation is transformed into a set of algebraic equations which can be solved with *Successive Overrelaxation (SOR)* or *Conjugate Gradient* methods. The solution of these simultaneous equations is the approximate solution of the original boundary value problem for given boundary values. The treatment of non-linear problems is also possible but requires more sophisticated (implicit) iterative solution procedures.
For solving time-dependent problems, the variation of time has to be approximated by difference quotients too. These quotients are iterated together with the iteration of the positions. But although special alternative iteration techniques are available, especially for large scale problems and high frequency applications the Finite-Difference Time-Domain Method (FDTD) is preferred [Taflove/Hagness 2000]. This method will be discussed later.

II. FINITE ELEMENT METHODS

By the end of the 1950s, the finite element method was introduced, firstly in structural mechanics [Turner, Clough 1956]. The significant difference between the finite difference method and the finite element method is that, in this method, the domain is discretized by employing a set of small elements with different shapes and sizes. With this approach it is easy to solve a problem having complex geometry and different interfacial boundaries to a degree of accuracy. It seems to be one of the most efficient methods for the solution of electromagnetic field problems. As both the finite difference method and the finite element method are based on differential equations and domain discretization, they are called *differential methods* or *domain methods*.

III. INTEGRAL EQUATION METHODS

Almost in the same period volume integral equation methods were developed for solving static magnetic field and eddy current problems [Symm 1963]. The volume integral equation method is based on the principle of superposition. First, the source area is subdivided into small areas; then the solution in terms of the sum of all such elements is sought. This method is simple to understand and easy to solve in the case of 2-dimensional problems. However, it is limited to just linear problems.
In order to reduce the region of discretization boundary integral equation methods were rapidly developed. The most typical of these based on the boundary integral equation is the boundary element method [Brebbia 1978]. The advantage of the boundary element method is that only the boundary values are treated as being unknowns and only the boundary of the solution domain is discretized. Hence, the post-data processing is therefore much easier than with the finite element method, especially in the case of 3-D problems.
However, integral equation methods have been considered much more in detail already in an earlier PhD seminar and further more information can be found there [Brauer 2004].

IV. FINITE DIFFERENCE TIME DOMAIN METHOD

The Finite-Difference Time-Domain (FDTD) method is the most direct solution of Maxwell's equations possible. It is complete and "full wave", that is, there are no approximations that prevent a correct solution from being reached. There is no preselection of modes or solution form. Boundary conditions are automatically satisfied. If a physical phenomenon exists, the FDTD method will include it in the solution.

While FDTD is based on a solution grid, this grid is fundamentally different than those used by other methods. The FDTD grid is composed of rectangular boxes. Each box edge is an electric field location, and the material for each mesh edge can be specified independently of other edges. The geometry is formed by assigning different materials to different mesh edges. This regular grid is chosen since making calculations for each grid element is extremely fast. This allows very precise approximations to the actual physical geometry.

The FDTD algorithm was first established by Yee as a three dimensional solution of Maxwell's curl equations. In principle, a volume of space containing any object or collection of objects is subjected to an electromagnetic disturbance, FDTD then solves for the fields throughout the volume as a function of time. Applications include, (but are not limited to): design of both discrete and integrated microwave circuits; radar cross section prediction; ionospheric and plasma scattering; integrated optics; interactions between biological bodies and EM fields; electromagnetic compatibility; antenna design etc. These applications can all be supported by the basic FDTD algorithm, requiring only different pre- and post-processing interfaces to obtain the final results.

Despite their complexity, the relevant parts of Maxwell's equations are straightforward to understand and to turn into a form appropriate for implementation on a computer.

If we are dealing with a region of space where there are no currents flowing and no isolated charges, the important parts of the Maxwell equations can be written simply as:

$$\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t} \quad \text{and} \quad \nabla \times \mathbf{H} = \varepsilon \frac{\partial \mathbf{E}}{\partial t}$$

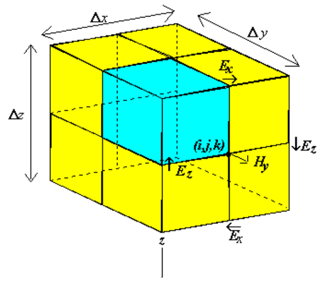

Fig. 4: Description of field quantities on a rectangular 3D grid (YEE cell)

To understand these operations, it is helpful to treat the fields as being discrete rather than continuous in space as shown in Figure 4. This shows a small cuboid or "Yee unit cell" of $\Delta x \cdot \Delta y \cdot \Delta z$ dimensions in which the **E** and the **H** field components are considered to be interlaced. With this picture in mind, the operation ∇x or "curl" is easily interpreted: the four E components surrounding the H_y located at point (i, j, k) in the figure trace out a clockwise path and therefore have some curl. If one E_x and one E_z were reversed in direction, the curl would be zero. The equations say that the H_y increases in response to a curl in this direction, with a constant of proportionality given by $1/\mu$. This constant specifies the magnetic permeability of the material at the location of the unit cell. A similar equation relates the rate of change of the E_s to the curl of the H components, this time with a constant of proportionality of $1/\varepsilon$, where ε is the permittivity of the material. If time is considered to be broken up into discrete steps Δt long, then the fields are simply increased or decreased from their previously calculated values by adding or subtracting an amount given by their curl calculated at the current time, then avoiding the partial differential operations. The E values are updated at $t = n\cdot\Delta t$ and the H values at $t = (n+1/2)\cdot\Delta t$. The curl operations are thereby reduced to a set of six simple algebraic expressions.

Magnetic Field Equations

$$H^{n+\frac{1}{2}}_{x(i,j,k)} = H^{n-\frac{1}{2}}_{x(i,j,k)} + \frac{\Delta t}{\mu\Delta z}\left(E^{n}_{y(i,j,k)} - E^{n}_{y(i,j,k-1)}\right) - \frac{\Delta t}{\mu\Delta y}\left(E^{n}_{z(i,j,k)} - E^{n}_{z(i,j-1,k)}\right)$$

$$H^{n+\frac{1}{2}}_{y(i,j,k)} = H^{n-\frac{1}{2}}_{y(i,j,k)} + \frac{\Delta t}{\mu\Delta x}\left(E^{n}_{z(i,j,k)} - E^{n}_{z(i-1,j,k)}\right) - \frac{\Delta t}{\mu\Delta z}\left(E^{n}_{x(i,j,k)} - E^{n}_{x(i,j,k-1)}\right)$$

$$H^{n+\frac{1}{2}}_{z(i,j,k)} = H^{n-\frac{1}{2}}_{z(i,j,k)} + \frac{\Delta t}{\mu\Delta y}\left(E^{n}_{x(i,j,k)} - E^{n}_{x(i,j-1,k)}\right) - \frac{\Delta t}{\mu\Delta x}\left(E^{n}_{y(i,j,k)} - E^{n}_{y(i-1,j,k)}\right)$$

Electric Field Equations

$$E^{n+1}_{x(i,j,k)} = E^{n}_{x(i,j,k)} + \frac{\Delta t}{\varepsilon\Delta y}\left(H^{n+\frac{1}{2}}_{z(i,j+1,k)} - H^{n+\frac{1}{2}}_{z(i,j,k)}\right) - \frac{\Delta t}{\varepsilon\Delta z}\left(H^{n+\frac{1}{2}}_{y(i,j,k+1)} - H^{n+\frac{1}{2}}_{y(i,j,k)}\right)$$

$$E^{n+1}_{y(i,j,k)} = E^{n}_{y(i,j,k)} + \frac{\Delta t}{\varepsilon\Delta z}\left(H^{n+\frac{1}{2}}_{x(i,j,k+1)} - H^{n+\frac{1}{2}}_{x(i,j,k)}\right) - \frac{\Delta t}{\varepsilon\Delta x}\left(H^{n+\frac{1}{2}}_{z(i+1,j,k)} - H^{n+\frac{1}{2}}_{z(i,j,k)}\right)$$

$$E^{n+1}_{z(i,j,k)} = E^{n}_{z(i,j,k)} + \frac{\Delta t}{\varepsilon\Delta x}\left(H^{n+\frac{1}{2}}_{y(i+1,j,k)} - H^{n+\frac{1}{2}}_{y(i,j,k)}\right) - \frac{\Delta t}{\varepsilon\Delta y}\left(H^{n+\frac{1}{2}}_{x(i,j+1,k)} - H^{n+\frac{1}{2}}_{x(i,j,k)}\right)$$

Notice that all of the operations are simple additions, subtractions and multiplications and are therefore simple to compute.

A real problem is solved simply by dividing it up into appropriately sized unit cells, each having a μ and ε value, setting initial values for all of the field components, then calculating the field equations iteratively for as long as the response is of interest. The behaviour of the field components over the problem space gives an essentially complete characterisation of the behaviour of the structure and can then be post processed in whatever way is appropriate for the application, giving information over a broad frequency range in a single run. This algorithm, as well as giving an understanding of the operations implied by Maxwell's equations, is a powerful and practical solution method known as the finite difference time domain method (FDTD) and was first described by Kane Yee in the year 1966 [Yee 1966].

FDTD is extremely general in the geometries it can analyze. Conductors, lossy dielectrics and magnetics, anisotropic materials, biological tissues, ferrites, and many other problems are extremely difficult for other methods, but are easy for FDTD. Furthermore, as problems become electrically large, the FDTD method quickly becomes more efficient in terms of computer time and memory than other methods since there is no matrix to fill and solve. FDTD can provide results for a wide spectrum of frequencies from just one calculation using transient pulse excitation and FFT.

At first glance, the stepped approximation that an FDTD grid makes to a slanted or curved geometry looks crude. But one must remember that the FDTD mesh size is 1/10 to 1/30 of a wavelength, sometimes smaller. So that while the mesh approximation may look less accurate than a smooth mesh produced by other methods, the actual deviation of the stepped FDTD mesh from the physical geometry will be less than for other methods since the FDTD grid size is so much smaller. The FDTD strategy is to use many very small mesh elements that can be computed quickly and with very little computer memory. In order to allow for large number of cells, it is extremely efficient in its use of memory. For example, for FDTD are required only about 30 MB of RAM per 1 million FDTD cells. This means that it is routine to make FDTD calculations with millions of cells in a few minutes. This is simply impossible with methods based on matrix fill and solve. In short, this is the power of FDTD.

SUMMARY

So far, any 2-D problem can be solved efficiently by one of these methods, and there are many well-designed commercial (and non-commercial) software packages for analyzing and designing purposes. However, for solving 3-D vector fields, especially in the case of problems containing non-linear material or having time-dependent solutions, efficient solution methods are still being developed. Another aspect which is of interest to engineers is to establish efficient software packages that can be used to model complex systems in designing practical electromagnetic devices.
As indicated, any such numerical method gives an approximate solution. To ensure this ultimately derives to real solution, the principle of error minimization should be observed.

CONCLUSIONS

Since electromagnetic analysis has been used as a prediction tool important for many branches of electrical engineering, electromagnetics will always remain important in electrical engineering technologies. However, the long and rich history of electromagnetics offers us a challenge on how we should educate the graduate students in this rapidly changing world. The total amount of knowledge cannot be taught to all the students within the short span of their graduate education. The important knowledge changes with changing times. Therefore, it is extremely important that we educate our students with the fundamental knowledge, so that they can read further independently if it needs be. Learning everything relevant to electromagnetics technology will entail a lifetime of learning. Also, it is important that we educate the students to become thinkers rather than one who regurgitates our knowledge mechanically. Furthermore, if we can educate a student who can empower an organization, the student will undoubtedly be more valuable to the society.

Since the role of a student changes from the university to the industry, it is important that students develop physical insight into their work as graduate students, so that they will become better system and design engineers in the industry. They will also become better problem solvers with better physical insight.

It is important that we bring the best and brightest, and the most creative people to work in our field. These new people will always create new frontier problems to work on. Consequently, there is no shortage of problems to work on and the field of electromagnetics will be constantly renewed.

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