Regular paper

THE USE OF CALCULATING FUNCTION FOR THE EVALUATION OF AXIAL FORCE BETWEEN TWO COAXIAL DISK COILS

Myrteza BRANESHI[1], Orion ZAVALANI[2] and Alfred PJETRI[3]

Abstract: In this paper, we present a simple and efficient approach for the calculation of the axial force between two coaxial circular disk coils in iron-free media. The core of this approach is the application of the calculating function of the axial force for the given configuration of coils, which represents a dimensionless primitive of force integral. The obtained expression of calculating function contains elliptic integrals of first and third kind and a term that must be numerically calculated by a standard procedure of the numerical integration. The calculating function, which depends on two variables, is tabulated, making possible the tabulated calculation of axial force. We compare the accuracy and the efficiency of the proposed approach with that of the equivalent filament method, where coils are presented by an entity of current-carrying filaments. The results obtained by two approaches are in very good agreement with each other.

Keywords: Axial force, Calculating function, Disk coil, Biot-Savart law, Filament method.

I. INTRODUCTION

Every magnetic field problem must solve different situations of tasks, depending on the scope that the problem has. One of those, the force evaluation, is still important to be studied in magnetic fields; it means that is still an open research area. Traditional methods elaborated to solve this sort of problems, stand on two main steps. The first step consists on the determination of magnetic field, in the domain with current. The second step consists on force evaluation [1-3]. So far the exactness of force calculation depends on the exactness of magnetic field calculation. Nowadays numerical methods as, finite element method (FEM) and the one of boundary elements (BEM) widely used for solving and giving solution of magnetic field problems, still have some problems related with the accuracy near sharp surfaces discontinuities unless a high density of elements is used [4, 5].

Exactly methods are successfully used for magnetic field problems solution. Their main advantage is the possibility to avoid the evaluation of magnetic field and direct calculation of unknown parameters or physical quantities [5, 6]. For the system of currents placed in iron free media, these methods are based on Biot-Savart law. An application area of exact methods is the different type of circular coils, because they are widely used in various electromagnetic devices (tubular linear motors, transformers and reactors, in plasma physics, MRI, antennas etc). Such methods are encountered in the calculation of the self and mutual inductances [7-13] where, according to our opinion, these methods are highly structured rather than those of calculating the force [14, 15].

In this paper, we represent a simple and efficient procedure for the calculation of axial force between two coaxial circular disk coils placed in an iron-free media. It consists in using the calculating function of axial force that represents a dimensionless primitive of the force integral. The expression of the calculating function is generated and it results in a form of two terms: one that expresses analytically by means of elliptic integrals and the other is a single integral with continuous kernel. To calculate this integral, one of the standard procedures of the numerical integration can be used. The tabulating form of the calculating function is presented due to the fact, that it is depended on two variables making possible the table calculation of force. To show the efficiency and accuracy of the proposed method, the obtained results are compared with the ones obtained by the equivalent filaments. Results stand very close to each other.

II. AXIAL FORCE MAGNITUDE

As the starting point in determination of the force between two circular coaxial disk coils is used the force between two circular coaxial filaments with current. Due to cylindrical symmetry only the axial component of the force exists. The force is repulsive if the currents are opposite in direction and attractive in the same direction. The magnitude of the force, in integral form, is as follows:

$$F_z = \frac{1}{2}\mu_0 I_R I_r \int_0^{2\pi} \frac{RrH\cos\theta d\theta}{\left(R^2 + r^2 + H^2 - 2Rr\cos\theta\right)^{3/2}} \qquad (1)$$

where μ_0 is the permeability of free space, I_R and I_r are currents of filaments with radii R and r respectively, and H is the distance between filament planes as shown in figure 1.

Force (1) is analytically expressed in terms of elliptic integrals [16]:

$$F_z = A_F\left[\frac{R^2 + r^2 + H^2}{(R-r)^2 + H^2}\mathbf{E}(k) - \mathbf{K}(k)\right] \qquad (2)$$

where

$$A_F = \frac{\mu_0 I_R I_r H}{\sqrt{(R+r)^2 + H^2}}$$

[1] Polytechnic University of Tirana, Department of Electrics, Sheshi Nënë Tereza, No. 4, Tirana, Albania, E-mail: mbraneshi@gmail.com
[2] Polytechnic University of Tirana, Department of Electrics, Sheshi Nënë Tereza, No. 4, Tirana, Albania, E-mail: orionzavalani@gmail.com
[3] Polytechnic University of Tirana, Department of Electrics, Sheshi Nënë Tereza, No. 4, Tirana, Albania, E-mail: alfredpjetri@yahoo.com

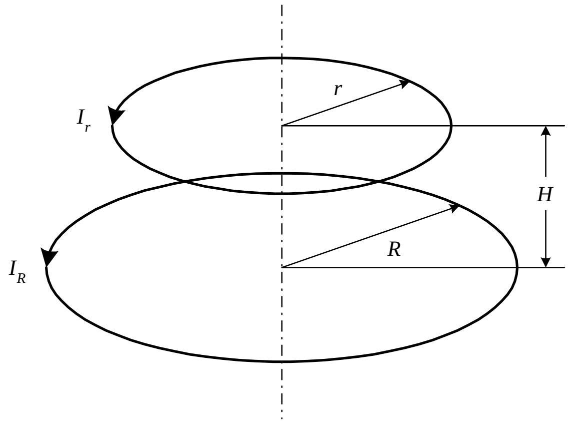

Fig. 1 *Two coaxial circular filaments with currents.*

and $\mathbf{K}(k)$ and $\mathbf{E}(k)$ are complete elliptic integrals of first and second kind respectively

$$\mathbf{K}(k) = \int_0^{\pi/2} \frac{d\beta}{\sqrt{1-k^2 \sin^2 \beta}} \tag{3}$$

$$\mathbf{E}(k) = \int_0^{\pi/2} \sqrt{1-k^2 \sin^2 \beta}\, d\beta \tag{4}$$

with the module k

$$k^2 = \frac{4Rr}{(R+r)^2 + H^2} \tag{5}$$

Figure 2 shows the dimensions and arrangement of the two coaxial disk-coils. In the case of the coils being constructed with thin wire and compactly wound we can suppose that:

– circular coaxial filaments can accurately model the spiral arrangement;

– the electrical currents can be considered uniformly distributed over the whole cross section of coils;

– due to the small sizes in axial direction of the coils ($h_1 << w_1 = R_2 - R_1$ and $h_2 << w_2 = r_2 - r_1$) can be considered the current distributions are concentrated in the coils central planes.

Denoting with N_1, N_2 and I_1, I_2 , number of turns in the windings and electrical current of each coil respectively, the current density in each coil is:

$$j_1 = \frac{N_1 I_1}{R_2 - R_1}, \qquad j_2 = \frac{N_2 I_2}{r_2 - r_1} \tag{6}$$

The calculation of the axial force between two coils is done by integrating the expression (1), where I_R is substituted by $j_1 dR$ and I_{re} is substituted by $j_2 dr$. The expression of axial force, in the integral form, results:

$$F = C_F \int_{R_1}^{R_2} dR \int_{r_1}^{r_2} dr \int_0^{2\pi} \frac{2\pi RrH \cos\theta d\theta}{\left(R^2 + r^2 + H^2 - 2Rr\cos\theta\right)^{3/2}} \tag{7}$$

where

$$C_F = \frac{\mu_0}{4\pi} \frac{N_1 N_2 I_1 I_2}{(R_2 - R_1)(r_2 - r_1)} \tag{8}$$

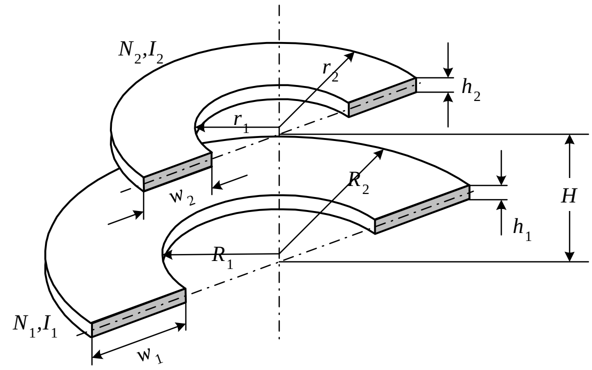

Fig. 2 *Two coaxial circular disk coils.*

III. PROPOSED METHOD

The proposed method of calculation of axial force consists in using its calculating function. This function represents a dimensionless primitive of integral of force (7). In [14], the calculating function is used to calculate the axial force between coaxial disk conductors, and in [17], it is used for the calculation of the force between magnets. Below we define the expression for computing the calculating function, which is a determinant function. Also we analyzed its attributes in some particular points of its variable. Calculating function depends on two variables; this makes possible its tabulation and consequently the tabulated calculation of the force. Also, for a particular region of its variables, an approximate expression is determined.

III.1. Determination of calculating function

Among the dimensionless primitives of the force integral (7) we define the calculating function of axial force of analyzed system of coils as follow:

$$\mathbf{F}_{\mathrm{DD}}(r,z) = \int_0^1 dR \int_0^r dt \int_0^{2\pi} \frac{2\pi Rtz \cos\theta d\theta}{\left(R^2 + t^2 + z^2 - 2Rt\cos\theta\right)^{3/2}} \tag{9}$$

One can see it is an odd function with respect to variable z. It also satisfies the following relation with regard to r

$$\mathbf{F}_{\mathrm{DD}}(r,z) = r^2 \mathbf{F}_{\mathrm{DD}}\left(\frac{1}{r}, \frac{z}{r}\right) \tag{10}$$

Axial force (7) is expressed by means of its calculating function (9) as follows

$$F = C_F \sum_{i=1}^{2} \sum_{j=1}^{2} (-1)^{i+j} R_i^2 \mathbf{F}_{\mathrm{DD}}\left(\frac{r_j}{R_i}, \frac{H}{R_i}\right) \tag{11}$$

where C_F is given by (8).

III.2. Expression for the calculating function $\mathbf{F}_{\mathrm{DD}}$

The calculating function (9) cannot be integrated in the framework of the elementary functions or special ones already known. It is represented in form of two terms, where one expresses analytically by means of complete elliptic integrals of first and third kind, and the other is a single integral with continuous kernel, which can be integrated with a standard procedure of numerical integration. The final expression of the function $\mathbf{F}_{\mathrm{DD}}$ results:

$$\mathbf{F}_{DD}(r,z) = 2\pi k\sqrt{r}z\Bigg\{-3\mathbf{K}(k)$$
$$+\frac{\sqrt{r^2+z^2}}{4r}\left[\left(\sqrt{r^2+z^2}-1\right)h_1\mathbf{\Pi}(h_1,k)\right.$$
$$\left.+\left(\sqrt{r^2+z^2}+1\right)h_2\mathbf{\Pi}(h_2,k)\right]$$
$$+\frac{\sqrt{1+z^2}}{4r}\left[\left(\sqrt{1+z^2}-r\right)h_3\mathbf{\Pi}(h_3,k)\right.$$
$$\left.\left.+\left(\sqrt{1+z^2}+r\right)h_4\mathbf{\Pi}(h_4,k)\right]\right\}$$
$$+2\pi z^2\int_0^{2\pi}\frac{1}{\sin\theta}\operatorname{arctg}\frac{r\sin^2\theta+z^2\cos\theta}{z\sin\theta X}d\theta \tag{12}$$

where

$$X=\sqrt{1+r^2+z^2-2r\cos\theta} \tag{13}$$

and Π is denoted complete elliptic integral of third kind

$$\mathbf{\Pi}(h,k)=\int_0^{\pi/2}\frac{d\beta}{\left(1+h\sin^2\theta\right)\sqrt{1-k^2\sin^2\beta}} \tag{14}$$

with module k

$$k^2=\frac{4r}{(1+r)^2+z^2} \tag{15}$$

and modules h_i, i = 1, 2, 3, 4

$$h_1=-\frac{2r}{\sqrt{r^2+z^2}+r}$$

$$h_2=\frac{2r}{\sqrt{r^2+z^2}-r}$$

$$h_3=-\frac{2}{\sqrt{1+z^2}+1}$$

$$h_4=\frac{2}{\sqrt{1+z^2}-1} \tag{16}$$

The kernel of integral is a continuous function in points sin θ = 0. In these points its values are:

$$\frac{X}{\cos\theta}$$

The behavior of the calculating function $\mathbf{F}_{DD}$ for $z \to 0$ requires a special analyze. At these points h_1, $h_3 \to -1$ and h_2, $h_4 \to \infty$. Thus, the behavior of function $\mathbf{F}_{DD}$ depends on the behavior of elliptic integral of third kind Π for these points [18]. So, for $h \to -1$ ($|h| < 1$) the behavior of Π is

$$-\frac{\pi}{2}\frac{h}{\sqrt{(1+h)(-h-k^2)}} \tag{17}$$

and for $h \to \infty$ is

$$\frac{k^2\mathbf{K}(k)}{k^2+h}+\frac{\pi}{2}\frac{\sqrt{h}}{\sqrt{(1+h)(h+k^2)}} \tag{18}$$

Based on (17) we can write

$$\lim_{z\to 0} h_1\mathbf{\Pi}(h_1,k)=-\frac{\pi r}{|z|k'}$$

$$\lim_{z\to 0} h_3\mathbf{\Pi}(h_3,k)=-\frac{\pi}{|z|k'}$$

where k' is complementary module

$$k'=\sqrt{1-k^2}$$

Also, based on (18)

$$\lim_{z\to 0} h_2\mathbf{\Pi}(h_2,k)=k^2\mathbf{K}(k)+\frac{\pi r}{|z|}$$

$$\lim_{z\to 0} h_4\mathbf{\Pi}(h_4,k)=k^2\mathbf{K}(k)+\frac{\pi}{|z|}$$

The values of calculating function $\mathbf{F}_{DD}(r, z)$ for z near zero can be obtained by substituting the above expressions in (12)

$$\mathbf{F}_{DD}(r,z\to 0\pm)=2\pi^2\operatorname{sgn}(z)\begin{cases}r^2 & 0<r<1\\ 1 & r>1\end{cases} \tag{19}$$

In appendix A, are given the values of function $\mathbf{F}_{DD}$ for $0 \le r \le 1$ and $0 \le z \le \infty$ Values of $\mathbf{F}_{DD}$ for $z < 0$ can be calculated by taking into consideration the fact that the calculating function is odd one, while for $r > 1$ values of $\mathbf{F}_{DD}$ can be calculated by using expression (10). For calculation of complete elliptic integrals are used the procedures given in [19, 20], and for the calculation of the integral in expression (12) is used fourth degree Newton-Cotes procedure [21].

III.3. An approximation expression

For the calculation of the calculating function in particular domains of r and z variables approximate expressions can be used. Following we will give an expression that is valid for $z \to \infty$ ($0 < r \le 1$). For $r > 1$ relation (10) it is recommended.

$$\mathbf{F}_{DD}(r,z)\cong\frac{2\pi^2r^3}{z^4}\operatorname{sgn}(z)\left[\frac{1}{3}-\frac{1}{2}\frac{A_2}{z^2}+\frac{5}{8}\frac{A_4}{z^4}-\frac{35}{48}\frac{A_6}{z^6}\right] \tag{20}$$

where

$$A_2 = 1 + r^2, \qquad A_4=1+\frac{63}{25}r^2+r^4,$$

$$A_6=1+\frac{162}{35}r^2+\frac{162}{35}r^4+r^6 \tag{21}$$

The use of the first two terms of (20) for $z \ge 10$ yields an accuracy up to 3 digits with the precise value, which increases up 4 digits for $z \ge 20$. The first three terms of (20) give an accuracy up to 4 digits for $z \ge 10$ while for $z \ge 20$ this goes up to 6 digits. The whole expression (20) gives an accuracy up to 3 digits for $z \geq 5$, which goes up to 5 digits for $z \geq 10$ and to 7 digits for $z \geq 20$.

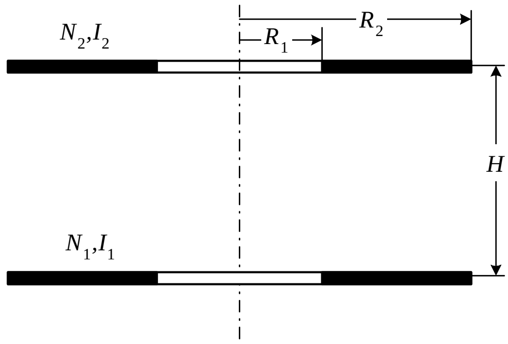

Fig. 3 *Coaxial geometrical identical circular disk coils.*

IV. CASE OF GEOMETRICAL IDENTICAL COILS

In the case of $R_1 = r_1$ and $R_2 = r_2$, as is shown in Figure 3, the geometrical configuration of the system of coils depends only upon the ratios R_1/R_2 and H/R_2. The expression (10) is given in the following form:

$$F = C_F R_2^2 \mathbf{F}_\mathrm{D}\left(\frac{R_1}{R_2}, \frac{H}{R_2}\right) = \frac{\mu_0}{4\pi} \frac{N_1 N_2 I_1 I_2}{(R_2 - R_1)^2} R_2^2 \mathbf{F}_\mathrm{D}\left(\frac{R_1}{R_2}, \frac{H}{R_2}\right) \quad (22)$$

where the function:

$$\mathbf{F}_\mathrm{D}(r, z) = r^2 \mathbf{F}_\mathrm{DD}\left(\frac{1}{r}, \frac{z}{r}\right) + \mathbf{F}_\mathrm{DD}(1, z) - 2\mathbf{F}_\mathrm{DD}(r, z) \quad (23)$$

can be used as a calculating function of the axial force between geometrical identical coils. The values of the function $\mathbf{F}_\mathrm{D}$ are shown in Appendix B for $0 \leq r \leq 1$ and $0 \leq z \leq \infty$.

V. EQUIVALENT FILAMENT METHOD

According to this method, each coils is divided in a determined number of elementary coils, where current-carrying filaments represent each of them. The force between two initial coils is the sum of forces between respective filaments

$$F = \sum_{i=1}^{m_1} \sum_{j=1}^{m_2} F_{ij} \quad (24)$$

where with m_1 and m_2 denote the numbers of filaments of the first and second coils respectively, while F_{ij} represents the force between the *i*-th filament of the first coil and *j*-th filament of the second coil. Based on (2), it is

$$F_{ij} = A_{Fij}\left[\frac{R_{Ki}^2 + r_{Kj}^2 + H^2}{(R_{Ki} - r_{Kj})^2 + H^2} \mathbf{E}(k_{ij}) - \mathbf{K}(k_{ij})\right] \quad (25)$$

where

$$A_{Fij} = \frac{\mu H I_{1i} I_{2j}}{\sqrt{(R_{Ki} + r_{Kj})^2 + H^2}}$$

and module of the elliptic integrals k_{ij}, according to (5) results

$$k_{ij}^2 = \frac{4 R_{Ki} r_{Kj}}{(R_{Ki} + r_{Kj})^2 + H^2} \quad (26)$$

Current I_{1i}, I_{2j} depends upon the way of coil division. Due to fact that the distributions of current density over cross-sections of coils are uniform we have apply uniform division, where elementary coils have the equal area of cross-section, as is shown in Figure 4. For this division we have:

$$I_{1i} = \frac{N_1 I_1}{m_1}, \qquad I_{2j} = \frac{N_2 I_2}{m_2}$$

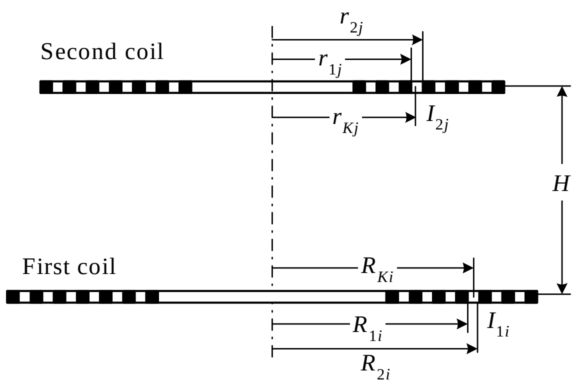

Fig. 4 *Division of disk coils in set of elementary coils for equivalent filament method.*

With regard to the radii of filaments, R_{Ki} and r_{Kj}, two approximations [22] are being analyzed:

– the filament is placed at the center of the cross section of elementary coils (arithmetical approximation)

$$R_{Ki} = \frac{1}{2}(R_{1i} + R_{2i}), \qquad r_{Kj} = \frac{1}{2}(r_{1j} + r_{2j}) \quad (27)$$

– the filament is placed at the geometrical center of the cross section (geometrical approximation)

$$R_{Ki} = \sqrt{R_{1i} R_{2i}}, \qquad r_{Kj} = \sqrt{r_{1j} r_{2j}} \quad (28)$$

where R_{1i}, R_{2i} are inner and outer radii respectively of the *i*-th elementary coil in first coil, and r_{1j}, r_{2j} are items of the *j*-th elementary coil in second coil. Their values can be calculated by following expressions:

$$R_{1i} = R_1 + (i-1)\frac{R_2 - R_1}{m_1} \qquad R_{2i} = R_1 + i\,\frac{R_2 - R_1}{m_1}$$

$$r_{1j} = r_1 + (j-1)\frac{r_2 - r_1}{m_2} \qquad r_{2j} = r_1 + j\,\frac{r_2 - r_1}{m_2}$$

VI. NUMERICAL EXAMPLES

To illustrate the above proposed method it is applied to calculate the force between two coaxial circular disk coils with the following data: R_1 = 16 cm, R_2 = 28 cm, N_1 = 100, I_1 = 10 A, r_1 = 11 cm, r_2 = 26 cm, N_2 = 100, I_2 = 10 A, where their planes are H = 5 cm distant from each other.

According to these data, from (8) we have

$$C_F = 5.555556$$

Values of function $\mathbf{F}_{\mathrm{DD}}$, satisfying the conditions of expression (11), are calculated by expression (12) and by using of the calculating tables (Appendix A). In case of values not present in the table a four-point interpolation is used

$$\mathbf{F}_{\mathrm{DD}}(r,z) = A_1\mathbf{F}_{\mathrm{DD}}(r_1,z_1) + A_2\mathbf{F}_{\mathrm{DD}}(r_2,z_1) + A_3\mathbf{F}_{\mathrm{DD}}(r_2,z_2) + A_4\mathbf{F}_{\mathrm{DD}}(r_1,z_2) \tag{29}$$

where $r_1 < r < r_1 + h_r = r_2$ and $z_1 < z < z_1 + h_z = z_2$ are the nearest table values of the given point, and h_r and h_z are the steps of table value, according to r and z respectively. A_i constants are calculated by means of these expressions

$$A_1 = \frac{r_2 - r}{h_r}\frac{z_2 - z}{h_z},$$

$$A_2 = \frac{r - r_1}{h_r}\frac{z_2 - z}{h_z},$$

$$A_3 = \frac{r - r_1}{h_r}\frac{z - z_1}{h_z},$$

$$A_4 = \frac{r_2 - r}{h_r}\frac{z - z_1}{h_z}. \tag{30}$$

The results are shown in Table I.

Table I

Calculation of axial force by means of its calculating function

$\frac{r_j}{R_i}$	$\frac{H}{R_i}$	$\mathbf{F}_{\mathrm{DD}}(r,z)$ according to (11)	$\mathbf{F}_{\mathrm{DD}}(r,z)$ according to (29)	Error [%]
0.6875	0.3125	3.589929	3.600556	–0.296
*1.625	*0.3125	*10.571442	*10.606614	–0.333
0.615385	0.192308	4.003386	4.016706	
0.392857	0.178571	1.427914	1.440516	–0.883
0.928571	0.178571	9.646443	9.668760	–0.231
F [N]		2.586693	2.587440	–0.029

Values marked with (*) are calculated by the expression (10).

It is obvious that the accurate values, as well as the interpolated one are in very good agreement with each other.

Table II shows the values of the force calculated according to the equivalent filament method for various numbers of filaments. Two filament radius approximations, arithmetical and geometrical one, are being compared. The error in computing of the force is given in table, also. The value of formula (11) is considered as an accurate value.

As a second example is calculated the axial force between two identical coaxial circular coils having the data: $R_1 = r_1 = 12$ cm, $R_2 = r_2 = 23$ cm, $N_1 = N_2 = 100$, $I_1 = I_2 = 10$ A, being in a distance $H = 2$ cm from each other. According to (8):

$$C_F R_2^2 = 0.437190$$

and according to (23):

$$\mathbf{F}_{\mathrm{D}} = 9.494209$$

The magnitude of the force, according to (22)

$$F = 4.150774 \text{ [N]}$$

The calculation according to the tables in the Appendix B for $\mathbf{F}_{\mathrm{D}}$, for $R_1/R_2 = 0.521739$ and $H/R_2 = 0.086957$, gives the value:

$$\mathbf{F}_{\mathrm{D}} = 9.545957$$

where four-point interpolation (29) is used. The magnitude of the force results:

$$F = 4.173398 \text{ [N]}$$

which differs with –0.545% from accurate value.

Table II

Axial force calculation by equivalent filament method

Number of divisions	Arithmetical approx.		Geometrical approx.	
	Force [N]	Error [%]	Force [N]	Error [%]
1	3.121006	–20.656	2.475323	4.305
2	2.624483	–1.461	2.528990	2.231
3	2.640000	–2.061	2.601582	–0.576
4	2.617600	–1.195	2.595998	–0.360
5	2.606183	–0.753	2.592440	–0.222
10	2.591557	–0.188	2.588149	–0.056
20	2.587907	–0.047	2.587057	–0.014
50	2.586887	–0.008	2.586751	–0.002
100	2.586741	–0.002	2.586707	–0.001

CONCLUSIONS

In this paper a simple and efficient approach of calculation of the force between two coaxial circular disk coils, with uniform distribution of current density, was being proposed. It consists in using calculating function of the force for the given configuration of coils. Computation of the calculating function is performed in analytical-numerical manner. The analytical term is expressed by complete elliptic integrals of first and third kind. The numerical term is a single integral with continuous kernel, which can be calculated with a standard procedure of numerical integration. Tabulating of the calculating function increases calculating speed and decreases its cost. The obtained results are in very good agreement with the equivalent filament method. Based on the results obtained from the considered example, we could come to a conclusion that the geometrical approximation of the filament radius gives much more accurate results.

APPENDIX A

THE VALUES OF FUNCTION $\mathbf{F}_{DD}(r, z)$ FOR $0 < r \le 1$ AND $0 \le z \le 1$

r	*z*										
	0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
0.00	0	0	0	0	0	0	0	0	0	0	0
0.05	0.049348	0.014525	0.007819	0.005215	0.003839	0.002984	0.002397	0.001968	0.001641	0.001382	0.001174
0.10	0.197392	0.094045	0.057366	0.039877	0.029874	0.023422	0.018909	0.015571	0.013003	0.010970	0.009327
0.15	0.444132	0.260124	0.174110	0.126361	0.096738	0.076751	0.062401	0.051613	0.043225	0.036540	0.031111
0.20	0.789568	0.518428	0.370184	0.278767	0.218037	0.175238	0.143634	0.119435	0.100387	0.085075	0.072568
0.25	1.233701	0.870800	0.651151	0.505190	0.402798	0.327813	0.270942	0.226580	0.191202	0.162499	0.138901
0.30	1.776529	1.317827	1.019520	0.810011	0.656569	0.540482	0.450295	0.378701	0.320875	0.273523	0.234327
0.35	2.418053	1.859542	1.476236	1.195340	0.982408	0.816878	0.685559	0.579650	0.493086	0.421575	0.361992
0.40	3.158273	2.495645	2.021302	1.661788	1.381559	1.158734	0.978775	0.831608	0.710042	0.608803	0.523934
0.45	3.997190	3.225573	2.654032	2.208849	1.853834	1.566202	1.330380	1.135232	0.972541	0.836092	0.721085
0.50	4.934802	4.048502	3.373124	2.835045	2.397798	2.038021	1.739350	1.489742	1.280039	1.103106	0.953282
0.55	5.971111	4.963287	4.176602	3.537899	3.010774	2.571544	2.203226	1.892964	1.630683	1.408313	1.219296
0.60	7.106115	5.968356	5.061647	4.313761	3.688694	3.162640	2.718073	2.341312	2.021316	1.749009	1.516858
0.65	8.339816	7.061504	6.024272	5.157453	4.425796	3.805470	3.278331	2.829723	2.447470	2.121340	1.842699
0.70	9.672212	8.239541	7.058748	6.061656	5.214105	4.492122	3.876610	3.351563	2.903351	2.520336	2.192614
0.75	11.103305	9.497610	8.156570	7.015898	6.042664	5.212133	4.503448	3.898556	3.381860	2.939988	2.561553
0.80	12.633094	10.827806	9.304528	8.004902	6.896465	5.951948	5.147127	4.460791	3.874707	3.373391	2.943774
0.85	14.261578	12.215974	10.480918	9.005984	7.755191	6.694525	5.793703	5.026933	4.372662	3.812991	3.333063
0.90	15.988759	13.633038	11.647948	9.985580	8.592361	7.419559	6.427543	5.584767	4.866021	4.250966	3.723032
0.95	17.814636	15.006208	12.738532	10.897164	9.376530	8.105077	7.032620	6.122144	5.345284	4.679705	4.107475
1.00	19.739209	16.126630	13.651660	11.688598	10.077030	8.730953	7.594613	6.628268	5.801975	5.092362	4.480748

r	*z*									
	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00
0.00	0	0	0	0	0	0	0	0	0	0
0.05	0.001004	0.000863	0.000744	0.000645	0.000561	0.000489	0.000427	0.000375	0.000329	0.000290
0.10	0.007979	0.006859	0.005922	0.005132	0.004462	0.003891	0.003403	0.002985	0.002624	0.002314
0.15	0.026642	0.022923	0.019803	0.017169	0.014934	0.013029	0.011399	0.010000	0.008796	0.007756
0.20	0.062227	0.053596	0.046340	0.040204	0.034991	0.030542	0.026733	0.023462	0.020645	0.018211
0.25	0.119296	0.102878	0.089040	0.077314	0.067336	0.058812	0.051508	0.045229	0.039817	0.035140
0.30	0.201604	0.174098	0.150850	0.131110	0.114283	0.099891	0.087543	0.076920	0.067756	0.059831
0.35	0.312004	0.269831	0.234085	0.203667	0.177693	0.155447	0.136338	0.119880	0.105671	0.093372
0.40	0.452404	0.391843	0.340371	0.296476	0.258929	0.226723	0.199025	0.175144	0.154504	0.136624
0.45	0.623747	0.541068	0.470617	0.410412	0.358826	0.314512	0.276353	0.243415	0.214917	0.190205
0.50	0.826015	0.717602	0.625007	0.545726	0.477682	0.419146	0.368674	0.325056	0.287276	0.254479
0.55	1.058248	0.920718	0.803011	0.702050	0.615263	0.540497	0.475945	0.420090	0.371653	0.329557
0.60	1.318574	1.148896	1.003413	0.878424	0.770822	0.677994	0.597740	0.528209	0.467836	0.415303
0.65	1.604260	1.399872	1.224353	1.073334	0.943135	0.830654	0.733276	0.648793	0.575342	0.511343
0.70	1.911777	1.670708	1.463396	1.284769	1.130548	0.997123	0.881446	0.780945	0.693442	0.617092
0.75	2.236903	1.957884	1.717616	1.510300	1.331047	1.175731	1.040871	0.923522	0.821195	0.731775
0.80	2.574865	2.257428	1.983704	1.747169	1.542331	1.364559	1.209946	1.075187	0.957483	0.854458
0.85	2.920522	2.565071	2.258101	1.992399	1.761903	1.561508	1.386907	1.234452	1.101053	0.984086
0.90	3.268607	2.876433	2.537143	2.242911	1.987168	1.764386	1.569893	1.399738	1.250564	1.119516
0.95	3.613989	3.187230	2.817227	2.495656	2.215534	1.970979	1.757016	1.569427	1.404626	1.259556
1.00	3.951949	3.493475	3.094957	2.747728	2.444505	2.179138	1.946417	1.741913	1.561850	1.403000

THE VALUES OF FUNCTION $\mathbf{F}_{DD}(r, z)$ FOR $0 \le r \le 1$ AND $1 \le z \le \infty$ $(0 \le 1/z \le 1)$

r	1/*z*										
	1.00	0.95	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.55	0.50
0.00	0	0	0	0	0	0	0	0	0	0	0
0.05	0.000290	0.000255	0.000221	0.000190	0.000160	0.000133	0.000108	0.000086	0.000067	0.000051	0.000037
0.10	0.002314	0.002032	0.001764	0.001512	0.001277	0.001061	0.000865	0.000690	0.000536	0.000404	0.000293
0.15	0.007756	0.006812	0.005916	0.005073	0.004287	0.003563	0.002906	0.002317	0.001801	0.001358	0.000987
0.20	0.018211	0.016001	0.013902	0.011926	0.010083	0.008384	0.006840	0.005458	0.004245	0.003202	0.002330
0.25	0.035140	0.030890	0.026851	0.023045	0.019495	0.016221	0.013242	0.010574	0.008229	0.006212	0.004523
0.30	0.059831	0.052625	0.045771	0.039309	0.033276	0.027707	0.022637	0.018091	0.014091	0.010648	0.007760
0.35	0.093372	0.082180	0.071528	0.061476	0.052082	0.043404	0.035493	0.028394	0.022139	0.016748	0.012220
0.40	0.136624	0.120339	0.104826	0.090172	0.076464	0.063786	0.052215	0.041818	0.032646	0.024726	0.018065
0.45	0.190205	0.167676	0.146193	0.125878	0.106853	0.089236	0.073136	0.058648	0.045845	0.034773	0.025442
0.50	0.254479	0.224548	0.195976	0.168925	0.143560	0.120040	0.098513	0.079108	0.061931	0.047048	0.034480
0.55	0.329557	0.291097	0.254337	0.219491	0.186770	0.156383	0.128524	0.103368	0.081055	0.061682	0.045285
0.60	0.415303	0.367247	0.321258	0.277601	0.236544	0.198351	0.163273	0.131535	0.103325	0.078774	0.057946
0.65	0.511343	0.452723	0.396544	0.343133	0.292819	0.245931	0.202782	0.163659	0.128804	0.098396	0.072529
0.70	0.617092	0.547059	0.479842	0.415830	0.355423	0.299019	0.247003	0.199731	0.157513	0.120583	0.089081
0.75	0.731775	0.649627	0.570654	0.495313	0.424078	0.357424	0.295816	0.239690	0.189432	0.145347	0.107627
0.80	0.854458	0.759656	0.668357	0.581092	0.498413	0.420877	0.349039	0.283424	0.224504	0.172665	0.128172
0.85	0.984086	0.876262	0.772228	0.672590	0.577979	0.489045	0.406434	0.330773	0.262632	0.202492	0.150702
0.90	1.119516	0.998482	0.881469	0.769157	0.662265	0.561535	0.467716	0.381540	0.303690	0.234756	0.175186
0.95	1.259556	1.125299	0.995226	0.870095	0.750711	0.637912	0.532557	0.435492	0.347524	0.269363	0.201574
1.00	1.403000	1.255676	1.112623	0.974674	0.842723	0.717709	0.600600	0.492370	0.393953	0.306199	0.229803

r	1/*z*									
	0.45	0.40	0.35	0.30	0.25	0.20	0.15	0.10	0.05	0.00
0.00	0	0	0	0	0	0	0	0	0	0
0.05	0.000026	0.000017	0.000010	0.000006	0.000003	0.000001	0.000000	0.000000	0.000000	0
0.10	0.000204	0.000135	0.000083	0.000047	0.000023	0.000010	0.000003	0.000001	0.000000	0
0.15	0.000687	0.000453	0.000279	0.000158	0.000079	0.000033	0.000011	0.000002	0.000000	0
0.20	0.001622	0.001070	0.000660	0.000373	0.000187	0.000079	0.000026	0.000005	0.000000	0
0.25	0.003152	0.002082	0.001285	0.000726	0.000365	0.000155	0.000050	0.000010	0.000001	0
0.30	0.005414	0.003579	0.002211	0.001251	0.000629	0.000267	0.000087	0.000017	0.000001	0
0.35	0.008535	0.005649	0.003494	0.001979	0.000996	0.000423	0.000138	0.000028	0.000002	0
0.40	0.012635	0.008374	0.005186	0.002942	0.001482	0.000630	0.000205	0.000041	0.000003	0
0.45	0.017821	0.011829	0.007338	0.004168	0.002102	0.000894	0.000292	0.000059	0.000004	0
0.50	0.024191	0.016085	0.009994	0.005686	0.002872	0.001224	0.000399	0.000081	0.000005	0
0.55	0.031831	0.021204	0.013199	0.007523	0.003806	0.001624	0.000531	0.000107	0.000007	0
0.60	0.040811	0.027241	0.016990	0.009703	0.004918	0.002101	0.000688	0.000139	0.000009	0
0.65	0.051191	0.034243	0.021404	0.012248	0.006220	0.002662	0.000873	0.000177	0.000011	0
0.70	0.063017	0.042251	0.026469	0.015181	0.007726	0.003313	0.001088	0.000221	0.000014	0
0.75	0.076320	0.051295	0.032214	0.018519	0.009445	0.004058	0.001335	0.000271	0.000017	0
0.80	0.091120	0.061400	0.038659	0.022280	0.011390	0.004904	0.001616	0.000329	0.000021	0
0.85	0.107423	0.072582	0.045823	0.026478	0.013570	0.005856	0.001933	0.000394	0.000025	0
0.90	0.125223	0.084848	0.053718	0.031126	0.015994	0.006918	0.002288	0.000467	0.000030	0
0.95	0.144502	0.098199	0.062354	0.036234	0.018669	0.008095	0.002683	0.000548	0.000035	0
1.00	0.165231	0.112629	0.071736	0.041811	0.021604	0.009392	0.003120	0.000639	0.000041	0

APPENDIX B

THE VALUES OF FUNCTION $\mathbf{F}_D(r, z)$ FOR $0 < r \le 1$ AND $0 \le z \le 1$

r	z										
	0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
0.00	19.739209	16.126630	13.651660	11.688598	10.077030	8.730953	7.594613	6.628268	5.801975	5.092362	4.480748
0.05	19.689861	16.101087	13.636597	11.678319	10.069405	8.725009	7.589830	6.624338	5.798698	5.089600	4.478401
0.10	19.541817	15.983347	13.550957	11.614118	10.019579	8.685235	7.557401	6.597477	5.776185	5.070562	4.462188
0.15	19.295077	15.762386	13.370352	11.467442	9.899702	8.586314	7.474983	6.528220	5.717566	5.020646	4.419467
0.20	18.949640	15.439011	13.090521	11.228843	9.697076	8.414186	7.328441	6.403095	5.610392	4.928564	4.340118
0.25	18.505508	15.014845	12.711981	10.896561	9.407631	8.163015	7.110853	6.214683	5.447189	4.787076	4.217310
0.30	17.962680	14.491549	12.236634	10.471843	9.031543	7.831848	6.820293	5.960329	5.224819	4.592783	4.047550
0.35	17.321156	13.870799	11.666988	9.957394	8.571281	7.422667	6.458152	5.640836	4.943565	4.345527	3.830338
0.40	16.580935	13.154359	11.006009	9.356894	8.030832	6.939453	6.028185	5.259594	4.606372	4.047796	3.567715
0.45	15.742019	12.344183	10.257178	8.674934	7.415489	6.387841	5.536067	4.822087	4.218380	3.704286	3.263884
0.50	14.804407	11.442541	9.424669	7.917161	6.731927	5.775099	4.989283	4.335716	3.786681	3.321628	2.924933
0.55	13.768098	10.452199	8.513635	7.090609	5.988491	5.110337	4.397236	3.809802	3.320242	2.908258	2.558661
0.60	12.633094	9.376685	7.530659	6.204218	5.195702	4.404925	3.771536	3.255743	2.829948	2.474365	2.174467
0.65	11.399393	8.220717	6.484479	5.269672	4.367030	3.673101	3.126422	2.687266	2.328706	2.031886	1.783266
0.70	10.066996	6.990937	5.387179	4.302694	3.520024	2.932815	2.479301	2.120708	1.831548	1.594485	1.397396
0.75	8.635904	5.697271	4.256292	3.325130	2.677936	2.206782	1.851328	1.575264	1.355661	1.177465	1.030461
0.80	7.106115	4.355718	3.118681	2.368302	1.871910	1.523630	1.267850	1.073033	0.920239	0.797533	0.697072
0.85	5.477630	2.994747	2.018134	1.478228	1.143545	0.918746	0.758396	0.638658	0.546027	0.472351	0.412446
0.90	3.750450	1.672649	1.030565	0.722597	0.546637	0.433865	0.355664	0.298301	0.254442	0.219837	0.191855
0.95	1.924573	0.535129	0.290428	0.194953	0.144798	0.113912	0.092951	0.077771	0.066258	0.057222	0.049942
1.00	0	0	0	0	0	0	0	0	0	0	0

r	z									
	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00
0.00	3.951949	3.493475	3.094957	2.747728	2.444505	2.179138	1.946417	1.741913	1.561850	1.403000
0.05	3.949943	3.491751	3.093468	2.746439	2.443384	2.178161	1.945562	1.741164	1.561191	1.402420
0.10	3.936058	3.479803	3.083147	2.737490	2.435601	2.171371	1.939623	1.735953	1.556608	1.398379
0.15	3.899334	3.448116	3.055712	2.713664	2.414848	2.153246	1.923751	1.722019	1.544344	1.387559
0.20	3.830766	3.388706	3.004106	2.668723	2.375618	2.118918	1.893641	1.695547	1.521016	1.366954
0.25	3.724020	3.295771	2.923054	2.597905	2.313620	2.064534	1.845837	1.653438	1.483845	1.334071
0.30	3.575631	3.165961	2.809373	2.498219	2.226075	1.987524	1.777974	1.593526	1.430850	1.287102
0.35	3.384845	2.998340	2.662011	2.368545	2.111832	1.886737	1.688924	1.514714	1.360979	1.225044
0.40	3.153292	2.794174	2.481917	2.209569	1.971357	1.762461	1.578827	1.417033	1.274173	1.147773
0.45	2.884671	2.556675	2.271851	2.023636	1.806629	1.616351	1.449062	1.301619	1.171366	1.056050
0.50	2.584489	2.290766	2.036170	1.814575	1.620991	1.451315	1.302148	1.170651	1.054438	0.951494
0.55	2.259861	2.002874	1.780635	1.587520	1.418996	1.271377	1.141632	1.027249	0.926130	0.836511
0.60	1.919366	1.700764	1.512235	1.348732	1.206240	1.081525	0.971953	0.875360	0.789949	0.714218
0.65	1.572909	1.393381	1.239018	1.105439	0.989201	0.887559	0.798304	0.719631	0.650055	0.588340
0.70	1.231588	1.090688	0.969924	0.865659	0.775073	0.695941	0.626490	0.565284	0.511148	0.463110
0.75	0.907506	0.803471	0.714587	0.638019	0.571599	0.513634	0.462786	0.417981	0.378345	0.343161
0.80	0.613514	0.543102	0.483120	0.431557	0.386889	0.347941	0.313788	0.283697	0.257072	0.233426
0.85	0.362867	0.321236	0.285860	0.255502	0.229232	0.206340	0.186272	0.168588	0.152937	0.139030
0.90	0.168787	0.149469	0.133084	0.119040	0.106895	0.096316	0.087041	0.078867	0.071628	0.065193
0.95	0.043955	0.038949	0.034708	0.031074	0.027933	0.025196	0.022796	0.020680	0.018805	0.017136
1.00	0	0	0	0	0	0	0	0	0	0

THE VALUES OF FUNCTION $\mathbf{F}_D(r, z)$ FOR $0 \le r \le 1$ AND $1 \le z \le \infty$ $(0 \le 1/z \le 1)$

r	$1/z$										
	1.00	0.95	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.55	0.50
0.00	1.403000	1.255676	1.112623	0.974674	0.842723	0.717709	0.600600	0.492370	0.393953	0.306199	0.229803
0.05	1.402420	1.255166	1.112180	0.974295	0.842403	0.717442	0.600383	0.492197	0.393819	0.306098	0.229730
0.10	1.398379	1.251618	1.109099	0.971654	0.840171	0.715588	0.598872	0.490992	0.392882	0.305392	0.229217
0.15	1.387559	1.242109	1.100837	0.964566	0.834178	0.710605	0.594807	0.487748	0.390361	0.303490	0.227833
0.20	1.366954	1.223983	1.085070	0.951026	0.822718	0.701065	0.587016	0.481525	0.385517	0.299832	0.225170
0.25	1.334071	1.195013	1.059835	0.929322	0.804320	0.685727	0.574470	0.471488	0.377691	0.293914	0.220853
0.30	1.287102	1.153558	1.023656	0.898147	0.777844	0.663610	0.556343	0.456956	0.366337	0.285309	0.214563
0.35	1.225044	1.098673	0.975654	0.856694	0.742558	0.634065	0.532071	0.437449	0.351058	0.273699	0.206055
0.40	1.147773	1.030181	0.915612	0.804716	0.698201	0.596826	0.501393	0.412724	0.331636	0.258897	0.195176
0.45	1.056050	0.948691	0.844002	0.742564	0.645016	0.552048	0.464392	0.382809	0.308059	0.240869	0.181879
0.50	0.951494	0.855585	0.761979	0.671184	0.583760	0.500317	0.421509	0.348020	0.280544	0.219751	0.166245
0.55	0.836511	0.752961	0.671348	0.592099	0.515693	0.442653	0.373546	0.308971	0.249542	0.195863	0.148487
0.60	0.714218	0.643572	0.574505	0.507366	0.442550	0.380490	0.321661	0.266571	0.215746	0.169713	0.128964
0.65	0.588340	0.530734	0.474369	0.419520	0.366496	0.315644	0.267347	0.222016	0.180088	0.142002	0.108180
0.70	0.463110	0.418244	0.374309	0.331509	0.290078	0.250278	0.212402	0.176771	0.143725	0.113618	0.086792
0.75	0.343161	0.310281	0.278056	0.246629	0.216166	0.186853	0.158901	0.132543	0.108031	0.085629	0.065598
0.80	0.233426	0.211315	0.189626	0.168451	0.147897	0.128084	0.109153	0.091259	0.074571	0.059272	0.045542
0.85	0.139030	0.126016	0.113240	0.100751	0.088612	0.076890	0.065667	0.055032	0.045086	0.035938	0.027699
0.90	0.065193	0.059165	0.053242	0.047446	0.041803	0.036345	0.031107	0.026133	0.021467	0.017161	0.013269
0.95	0.017136	0.015572	0.014034	0.012526	0.011056	0.009632	0.008262	0.006958	0.005731	0.004595	0.003564
1.00	0	0	0	0	0	0	0	0	0	0	0

r	$1/z$									
	0.45	0.40	0.35	0.30	0.25	0.20	0.15	0.10	0.05	0.00
0.00	0.165231	0.112629	0.071736	0.041811	0.021604	0.009392	0.003120	0.000639	0.000041	0
0.05	0.165180	0.112595	0.071715	0.041799	0.021598	0.009390	0.003119	0.000639	0.000041	0
0.10	0.164823	0.112360	0.071570	0.041717	0.021557	0.009372	0.003113	0.000637	0.000041	0
0.15	0.163860	0.111725	0.071178	0.041496	0.021446	0.009325	0.003098	0.000634	0.000041	0
0.20	0.162003	0.110499	0.070421	0.041068	0.021231	0.009234	0.003068	0.000628	0.000040	0
0.25	0.158990	0.108505	0.069189	0.040371	0.020880	0.009086	0.003020	0.000619	0.000040	0
0.30	0.154590	0.105588	0.067383	0.039346	0.020364	0.008866	0.002949	0.000604	0.000039	0
0.35	0.148623	0.101623	0.064921	0.037947	0.019658	0.008566	0.002850	0.000584	0.000037	0
0.40	0.140969	0.096522	0.061745	0.036136	0.018742	0.008175	0.002723	0.000559	0.000036	0
0.45	0.131582	0.090244	0.057823	0.033894	0.017605	0.007689	0.002564	0.000526	0.000034	0
0.50	0.120504	0.082807	0.053159	0.031218	0.016243	0.007105	0.002372	0.000488	0.000031	0
0.55	0.107867	0.074288	0.047796	0.028128	0.014664	0.006426	0.002148	0.000442	0.000028	0
0.60	0.093912	0.064838	0.041820	0.024671	0.012891	0.005660	0.001896	0.000391	0.000025	0
0.65	0.078987	0.054682	0.035366	0.020919	0.010959	0.004823	0.001618	0.000334	0.000021	0
0.70	0.063551	0.044126	0.028624	0.016981	0.008921	0.003936	0.001324	0.000274	0.000018	0
0.75	0.048178	0.033557	0.021838	0.012997	0.006849	0.003030	0.001021	0.000212	0.000014	0
0.80	0.033554	0.023449	0.015312	0.009144	0.004834	0.002145	0.000725	0.000150	0.000010	0
0.85	0.020475	0.014359	0.009410	0.005640	0.002992	0.001332	0.000452	0.000094	0.000006	0
0.90	0.009842	0.006927	0.004557	0.002741	0.001460	0.000652	0.000222	0.000046	0.000003	0
0.95	0.002653	0.001874	0.001238	0.000748	0.000400	0.000179	0.000061	0.000013	0.000001	0
1.00	0	0	0	0	0	0	0	0	0	0

REFERENCES

[1] O. Chadebec, J.L. Coulomb, F. Janet, “A review of magnetostatic moment method”, IEEE Trans. on Magnetics, Vol. 42, No. 4, 2006, 515-518

[2] E. Schmidt, H. de Gersen, T. Wailand, “Application of computationally efficient air-gap element within finite element analysis of magnetic bearing”, IEEE Trans. on Magnetics, Vol. 42, No. 4, 2006, 1263-1266

[3] V. Kim, D.K. Lien, “Effect of magnetitially induces vibration force in brushless permanent-magnet motors”, IEEE Trans. on Magnetics, Vol. 41, No. 6, 2005, 2164-2172

[4] J.T. Conway, “Exact solutions for magnetic fields of axisymetric solenoids and current distributions”, IEEE Trans. on Magnetics, Vol. 37, No. 4, 2001, 2977-2988

[5] B. Azzerboni, E. Cardelli, “Analytical evaluation of magnetic energy and stresses in 3_D conductor systems”, IEEE Trans. on Magnetics, Vol. 32, No. 2, 1996, 547-551

[6] R.T. Honjo, R.M. del Vecchio, “A program to compute magnetic fields, forces, and inductances due to solid rectangular conductors arbitrarily positioned in space”, IEEE Trans. on Magnetics, Vol. 22, No. 6, 1986, 1532-1535

[7] S. Babic, C. Akyel, “New analytic-numerical solutions for the mutual inductance of two coaxial circular coils with rectangular cross section in air”, IEEE Trans. on Magnetics, Vol. 42, No. 6, 2006, 1661-1669

[8] S. Babic, S. Salon, C. Akyel, “The mutual inductance of two thin coaxial disk coils in air”, IEEE Trans. on Magnetics, Vol. 40, No. 2, 2004, 822-825

[9] S. Babic, C. Akyel, S. Salon, “New procedures for calculation the mutual inductance of the system: filamentary circular coil-massive circular solenoid”, IEEE Trans. on Magnetics, Vol. 39, No. 3, 2003, 1131-1134

[10] C. Akyel, S. Babic, S. Kincic, “New and fast procedures for calculating the mutual inductance of coaxial circular coils (circular coil-disk coil)”, IEEE Trans. on Magnetics, Vol. 38, No. 5, 2002, 2367-2369

[11] S. Babic, C. Akyel, “Improvement in calculation of self- and mutual inductance of thin-wall solenoids and disk coils”, IEEE Trans. on Magnetics, Vol. 36, No. 4, 2000, 1970-1975

[12] K.B. Kim, E. Levi, Z. Zabar, L. Birenbaum, “Mutual inductance of noncoaxial circular coils with constant current density”, IEEE Trans. on Magnetics, Vol. 33, No. 5, 1997, 4303-4309

[13] D. Yu, K.S. Han, “Self-inductance of air-core coils with rectangular cross section”, IEEE Trans. on Magnetics, Vol. 23, No. 6, 1987, 3916-3921

[14] M. Braneshi, P. Cipo, L. Ahma, E. Hamiti, “Calculation of force between two coaxial disk-conductors by calculating functions”, J. of Applied Electromagnetism, Vol. 6, No. 2, 2004, 10-23, Greece

[15] K.B. Kim, E. Levi, Z. Zabar, L. Birenbaum, “Restoring force between two noncoaxial circular coils”, IEEE Trans. on Magnetics, Vol. 32, No. 2, 1996, 478-484

[16] L.A. Bessonov: “*Teoretiçeskie osnovi elektriçestva, elektromagnetiçeskoje polje*”, Gardariki, 2001, Moskva, Rusia.

[17] M. Braneshi, O. Zavalani, Y. Luga, “The calculation of levitation force between axially polarized magnetic disks by calculating function”, J. of Applied Electromagnetism, Vol. 6, No. 1, 2004, 1-12, Greece

[18] R.G. Selfridge, J.E. Maxfield: “*A table of the incomplete elliptic integral of the third kind*”, Dover Publication INC, New York, 1958

[19] L. Urankar, P. Henninger, “Compact extended algorithms for elliptic integrals in electromagnetic field and potential computations Part I: Elliptic integrals of the first and second kind with extended integration range”, IEEE Trans. on Magnetics, Vol. 27, No. 5, 1991, 4338-4342

[20] L. Urankar, P. Henninger, F.S. Nestel, “Compact extended algorithms for elliptic integrals in electromagnetic field and potential computations Part II: Elliptic integral of third kind with extended integration range”, IEEE Trans. on Magnetics, Vol. 30, No. 3, 1994, 1236-1241

[21] A. Angot: “*Complements de mathematiques*”, Paris, 1957

[22] W.G. Huley, M.C. Duffy, “Calculation of self and mutual impedances in planar magnetic structures”, IEEE Trans. on Magnetics, Vol. 31, No. 4, 1995, 2416-2422

Myrteza Braneshi was born in Elbasan, Albania, in 1958. He received the Dipl. Ing. degree from the Engineering Faculty, University of Tirana in 1983 and Ph.D. degree from the Electrical Engineering Faculty, Polytechnic University of Tirana in 1996.

From the 1983 he was with the Electrical Engineering Faculty of Polytechnic University of Tirana as an Assistant, a Lecturer and now, he held an Associate Professor position until 2005. His main research interests are in the development of numerical methods for the computation of electromagnetic fields, mathematical modeling stationary, quasi-stationary field, wave propagation and scattering, electromagnetic compatibility and health effects of electromagnetic fields on the human beings.