Regular paper

# DETERMINING STATIC TORQUE CHARACTERISTICS OF PERMANENT MAGNET STEP MOTOR

Miroslav BJEKIĆ[1]

**Abstract:** The paper analyzes three methods of calculating static torque of permanent magnet step motor. Analytical formula for calculating electromagnetic torque is derived. The obtained analytical equation is verified by comparing to the results obtained through FEM simulation.

**Keywords:** Static characteristics, Permanent magnet step motor, Finite element method.

## INTRODUCTION

In this paper three methods of calculating static torque characteristics are developed. These are as follows:

1. By means of solving entire magnetic circuit with the determined currents of all three phases and calculating electromagnetic torque in two ways: by using Maxwell's stress tensor and energy increment of magnetic field.
2. Static torque characteristic is represented as the sum of torques developed by each individual phase. In this case magnetic circuit is presumed to be linear.
3. By means of derivative of included magnetic fluxes of permanent magnets through each stator phase with respect to angle.

Approximately the same results are obtained in all three methods. Which of these will be applied depends on the complexity of geometric shape and magnet characteristics of each actual magnetic circuit.

## SOLVING THE ENTIRE MAGNETIC CIRCUIT BY USING FEM METHOD – THE APPLICATION OF MAXWELL'S STRESS TENSOR AND ENERGY INCREMENTS OF MAGNETIC FIELD (I METHOD)

The first method is comprehensive, because no simplifications are needed: magnetic field can be non-linear; stator phase currents have arbitrary values and are determined independently from one another [1, 2]. This method is also the most accurate, and can be used to check the correctness of other methods.

Two types of magnetic circuits are solved in this paper:

a) rotor with non-magnetic core (Fig. 1) – marked as PVC in the paper, and

b) rotor with magnetic core (Fig. 2) – marked as FE in the paper.

Figures 1 and 2 represent cross-section of three-phase four-pole step motor, with all materials and currents indicated for both motor types. Also, triangular mesh dividing magnetic field into more than 20,000 nodes and 45,000 elements is shown.

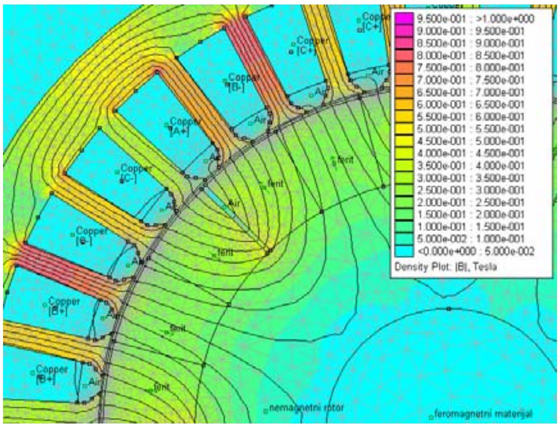

**Fig. 1** - *Rotor with non-magnetic core*

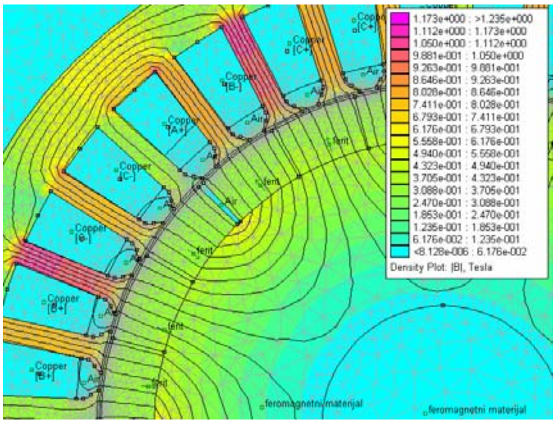

**Fig. 2** - *Rotor with magnetic core*

The simulation was carried out for the following three cases of non-magnetic and ferromagnetic rotor:

1. $I_1$=0,8 A, $I_2$=0 A, $I_3$=0 A
2. $I_1$=0,8 A, $I_2$= - 0,8 A, $I_3$=0 A
3. $I_1$=0,8 A, $I_2$= - 0,8 A, $I_3$= - 0,8 A

The rotor was rotated by $1^o$ (mechanical angle) at a time and electromagnetic torque was calculated for angles $0^o$ – $90^o$. In total, 540 simulations of magnetic circuit were carried out (2 types of rotors x 3 cases x 90 positions).

Figure 3 illustrates the calculated static torque characteristics when only the first stator phase is excited. Angle $0^o$ suits the first phase balance position.

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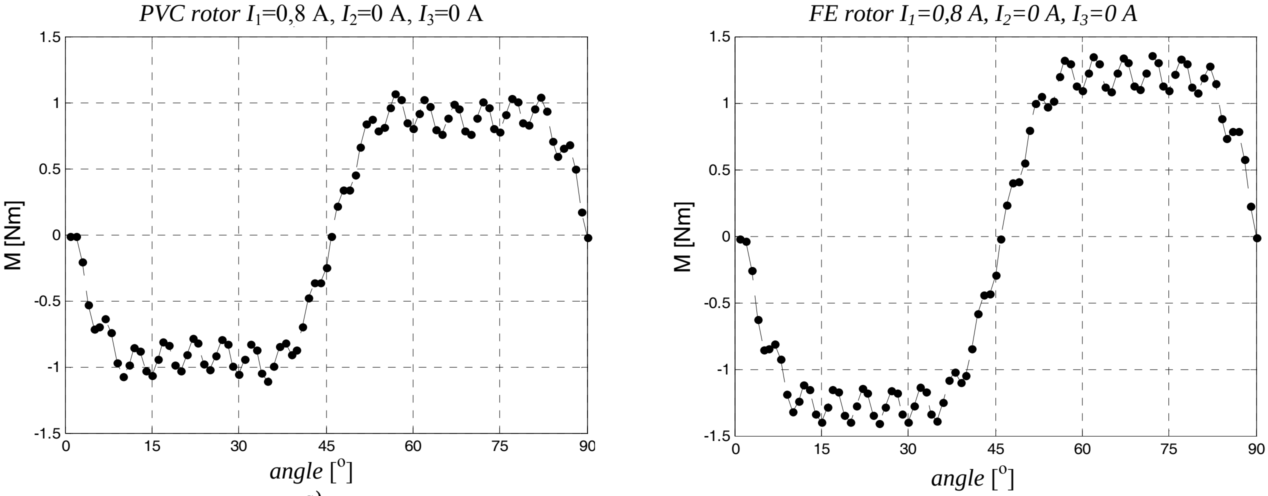
[1] University of Kragujevac, Technical faculty Cacak, Svetog Save 65, 32000 Cacak, Serbia, E-mail: mbjekic@ptt.yu

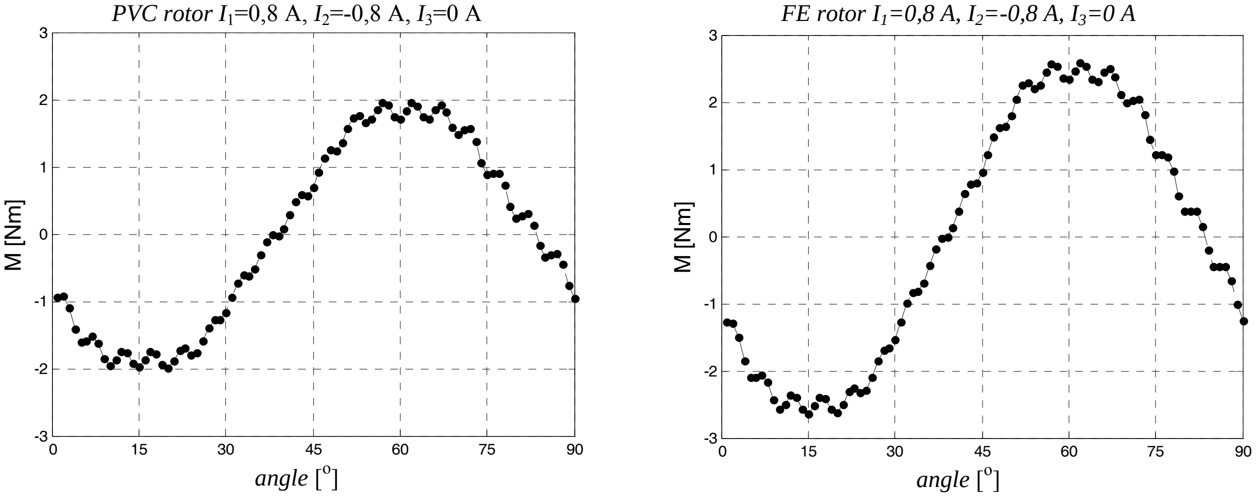
Figure 4 shows static torque characteristics when the first and second phases are excited simultaneously.

Figure 5 shows static torque characteristics when all three phases are excited simultaneously.



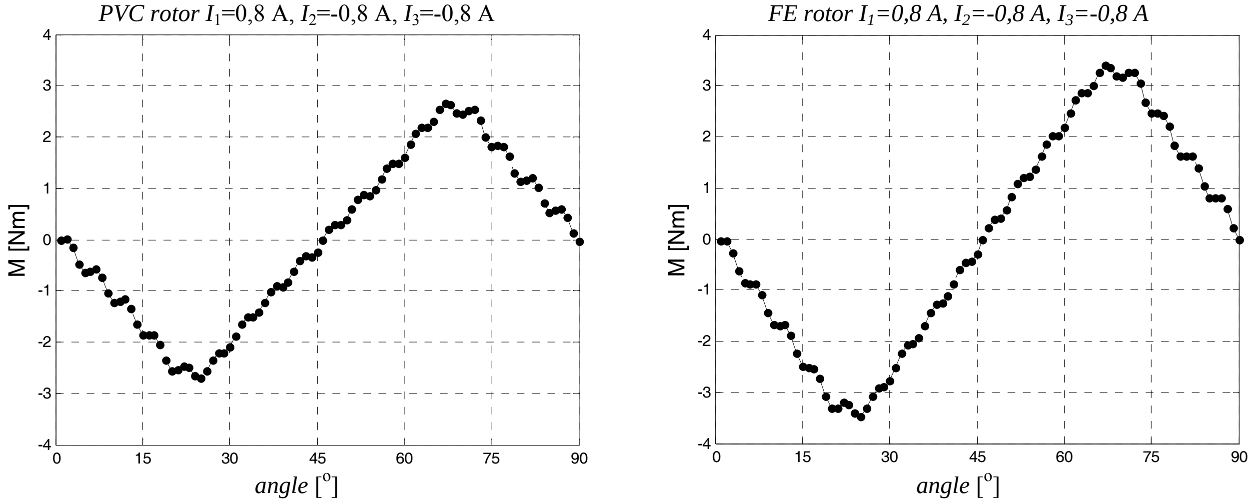
a) b)

**Fig. 3** - *Static characteristics, 1 phase excited*



a) b)

**Fig. 4** *Static characteristics, 2 phases excited simultaneously*



a) b)

**Fig. 5** – *Static characteristics, 3 phases excited simultaneously*

The differences between the calculated values by using Maxwell’s stress tensor and energy increments of magnetic field were negligible. These values will be used further in the paper as most accurately calculated values as well as a measure for comparing with the values obtained by applying other methods.

## CALCULATING STATIC TORQUE CHARACTERISTICS AS THE SUM OF INDIVIDUAL TORQUES OF EACH PHASE (II METHOD)

If we presume that step motor magnetic circuit is in linear working regime, it is possible to calculate total static torque characteristics as the sum of torques (calculated according the method previously mentioned) developed by each individual phase.

$$M = M_{f1} + M_{f2} + M_{f3}$$

Figure 6 illustrates and compares the accurate and approximate, previously explained method for developing electromagnetic torque (FEMM values, approximate values).

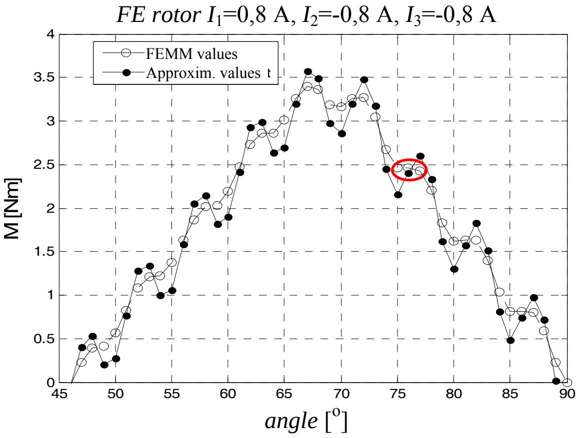


**Fig. 6** - *FE rotor, 3 phases excited*

There is the result deviation from the most accurate FEMM value when total torque is calculated as the sum of individual torques.

The reason is the following: the torque obtained by solving magnetic circuit of step motor consists of two components.

The first component is caused by current flow through the stator coils and by tendency towards concordance between magnetic fields of excited stator phases and magnetic field developed by permanent magnets (electromagnetic torque).

The second component is the torque occurring as the result of stator grooves (reluctance torque).

In other words, Figure 6 shows reluctance torque obtained as the sum of individual torques where reluctance torque is calculated as many times as there are addends. It means that the total torque developed by $N$ of exciting currents should be calculated according to the following equation:

$$M = \sum_{i=1}^{N \le 3} M_{fi} - (N-1) M_{rel} \qquad (1)$$

Figure 7 compares the calculated values according to the accurate method (magnetic circuit solved by means of FEMM method with all exciting currents) and the torque obtained as the sum of individual torques for each phase and reduced by multiple added reluctance torque.

The part of graph where differences between accurate and approximate values are considerable was zoomed.

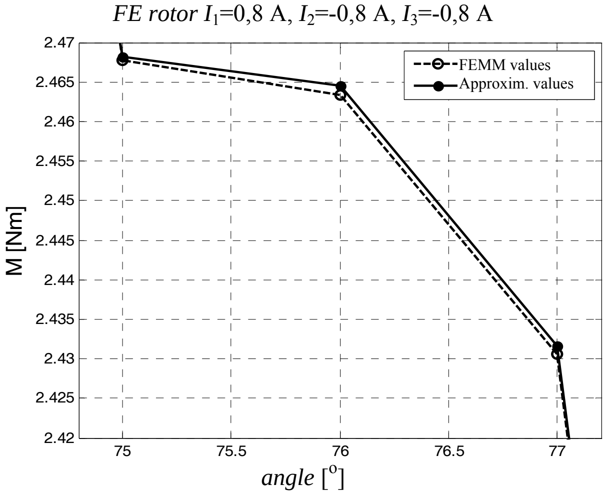


**Fig. 7** - *Accurate and approximate values of static characteristics*

The advantage of such a way of calculating electromagnetic torque is that it is not necessary to solve magnetic circuit by means of finite element method each time. Stator currents in transient working regime can have complex periodic shape on which developed static characteristics depends directly.

The practical question is: which values of exciting currents lead to the saturation of magnetic circuit, since the basic assumption for applying this method is that magnetic circuit is linear so that the superposition principle could be applied. That is the reason why the simulation of magnetic circuit was carried out for the currents for all stator phases. The currents changed at the intervals from negative to positive four-folded nominal value ($I_n$=0,8 A): $I_{i1}$= -3,2 A, $I_{i2}$= -1,6 A, $I_{i3}$= 0 A, $I_{i4}$= 1,6 A and $I_{i5}$= 3,2 A

The programme to calculate electromagnetic torque for $5^3$ =125 different combinations of currents for all three stator phases was created. For each of them magnetic circuit is solved and electromagnetic torque is calculated. The simulations for non-magnetic and magnetic rotors are carried out.

Figure 8 shows three-dimension graph illustrating the dependence of developed torques on stator currents of ferromagnetic rotor at the angle $\theta=0^\circ$. In this three-dimensional graph cross-section planes are selected when one of three stator currents is zero. In the graph we can notice that the torque does not depend on the current $I_1$ because the angle $\theta=0^\circ$ is the first phase balance position.

Figure 9 shows two-dimension illustration of ferromagnetic rotor's dependence of developed torque on two stator currents $M=f(I_2, I_3)$, where the current $I_1$=0. It can be noticed that, for the different rotor angles, the torque is changed depending on the changes of individual stator currents.

Figure 10 shows the difference between the calculated torque obtained by means of the most accurate FEMM method and the torque obtained by means of adding up individual torques. In this graph the extreme values can be read and maximum error can be calculated as a result of applying this simplified method.

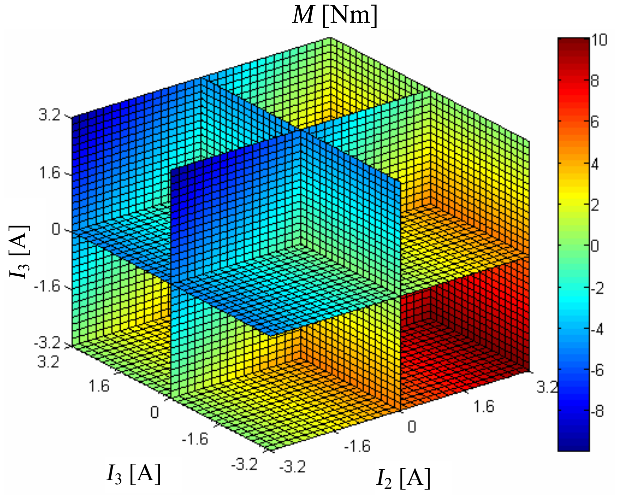


**Fig. 8** – *Dependence of electromagnetic torque on all stator phases for the rotor position* $\theta=0^{\circ}$

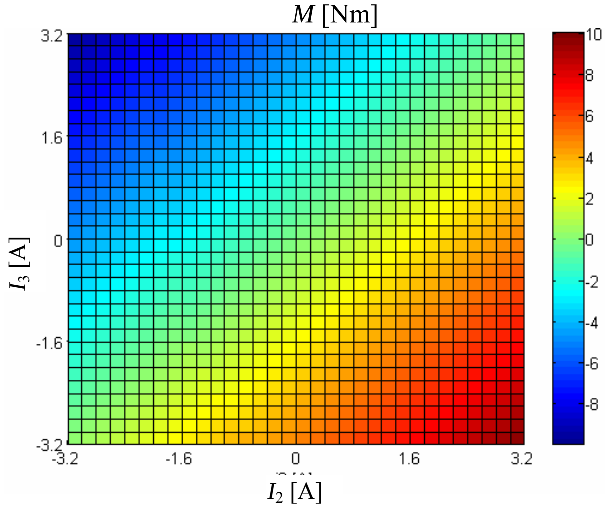


**Fig. 9** - *Dependence of electromagnetic torque on currents* $I_2$ *and* $I_3$ *at current* $I_1=0$ *for rotor position* $\theta=0^{\circ}$ *ΔM [Nm]*

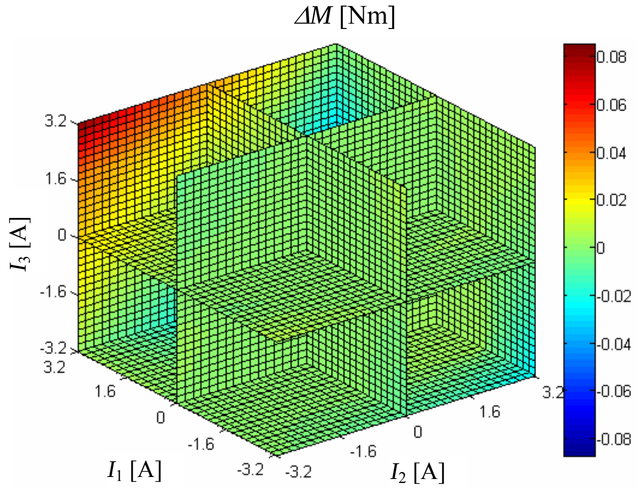


**Fig. 10** - *Differences of torques* $\Delta M$ *(accurate and approximate method)*

## CALCULATING STATIC TORQUE CHARACTERISTICS BY USING DERIVATIVE OF MAGNETIC FLUXES OF EACH PHASE WITH RESPECT TO ANGLE (III METHOD)

The next step in simplifying the method for calculating static characteristics is to determine proportionality factor between current intensity in the excited stator coil and developed electromagnetic torque.

$$M = \frac{1}{2} I_1 \frac{d\psi_{f1}}{d\theta} + \frac{1}{2} I_2 \frac{d\psi_{f2}}{d\theta} + \frac{1}{2} I_3 \frac{d\psi_{f3}}{d\theta} \tag{2}$$

That dependence is expressed by equation 2 and, dimensionally, this proportionality factor represents derivative of involved flux (flux in the stator coils originating from permanent magnets) of the phase with respect to angle whose component of developed electromagnetic torque is to be calculated.

It is necessary to solve step motor magnetic circuit when all currents of stator phases are zero, for different angles of rotor rotation. Figure 11 shows stator teeth numbered and phases marked, with current directions.

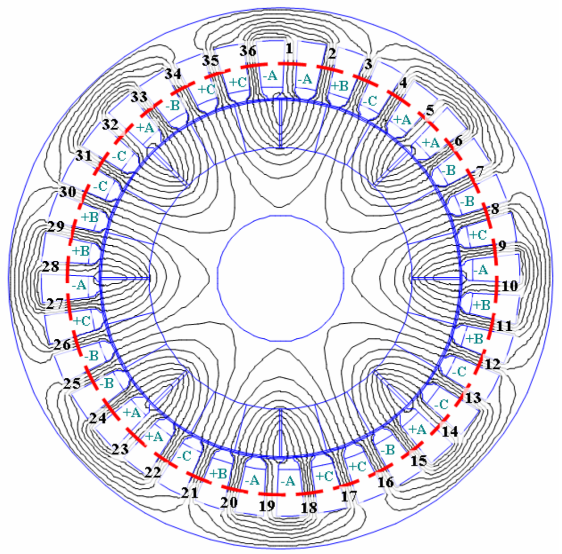


**Fig. 11** - *Cross-section of magnetic circuit with marked stator grooves and currents' directions*

Total involved flux of each phase is calculated as the sum of fluxes through stator teeth.

$$\begin{aligned}\psi_{f1} = {} & \psi_1 + 2\psi_2 + 2\psi_3 + 2\psi_4 + \psi_5 + \\ & + \psi_{10} + \psi_{11} + \psi_{12} + \psi_{13} + \psi_{14} + \\ & + \psi_{19} + 2\psi_{20} + 2\psi_{21} + 2\psi_{22} + \psi_{23} + \\ & + \psi_{28} + \psi_{29} + \psi_{30} + \psi_{31} + \psi_{32}\end{aligned} \tag{3}$$

$$\begin{aligned}\psi_{f2} = {} & \psi_7 + 2\psi_8 + 2\psi_9 + 2\psi_{10} + \psi_{11} + \\ & + \psi_{16} + \psi_{17} + \psi_{18} + \psi_{19} + \psi_{20} + \\ & + \psi_{25} + 2\psi_{26} + 2\psi_{27} + 2\psi_{28} + \psi_{29} + \\ & + \psi_{34} + \psi_{35} + \psi_{36} + \psi_1 + \psi_2\end{aligned} \tag{4}$$

$$\begin{aligned}\psi_{f3} = {} & \psi_{13} + 2\psi_{14} + 2\psi_{15} + 2\psi_{16} + \psi_{17} + \\ & + \psi_{23} + \psi_{24} + \psi_{25} + \psi_{26} + \psi_{27} + \\ & + \psi_{31} + 2\psi_{32} + 2\psi_{33} + 2\psi_{34} + \psi_{35} + \\ & + \psi_4 + \psi_5 + \psi_6 + \psi_7 + \psi_8\end{aligned} \tag{5}$$

Since winding of each stator phase contains $N$ coils, developed electromagnetic torque can be calculated by means of calculated fluxes $\psi_{f1}$, $\psi_{f2}$ and $\psi_{f3}$:

$$M = \sum_{i=1}^{3} C_i \, i_i + M_{\mathrm{rel}} \tag{6}$$

$$C_i(\theta) = N \frac{180}{\pi} \left( \psi_{f\,i}(\theta'') - \psi_{f\,i}(\theta') \right) \tag{7}$$

Quotient $180/\pi$ appears because numerical increment of flux per angle $\Delta\psi/\Delta\theta$ is calculated, and rotor position is changed by 1° at a time.

Figure 12 shows individual torques for each phase and reluctance torques at current intensity $I_i$=0,8 A for both rotor types.

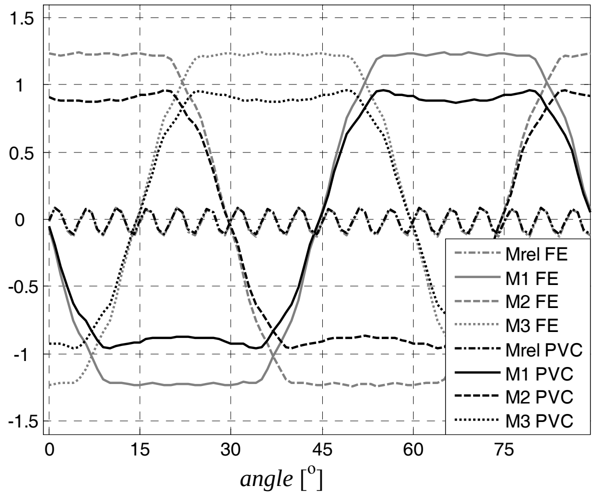

*PVC and FE rotor* $I_i$=0,8 A

**Fig. 12** – *Torques for each phase of PVC and FE rotor*

Figures 13, 14 and 15 show the value of resulting static torque characteristics developed by means of solving magnetic circuit using finite element method as well as the values of components of the first phase electromagnetic torques, reluctance torque and their sum for FE rotor. The figures illustrate considerable concordance between most accurately and approximately calculated static characteristics.

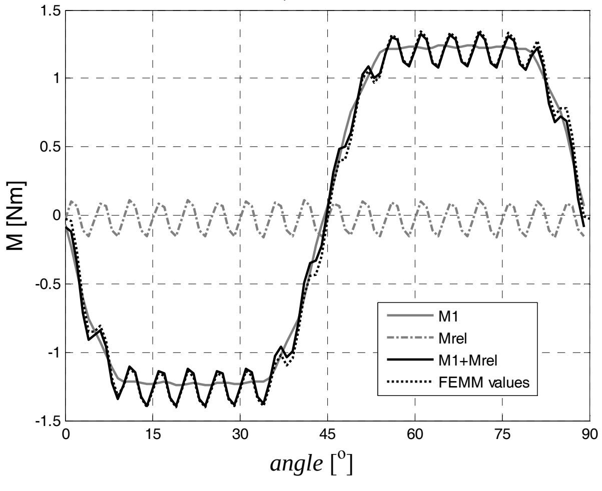

*FE rotor* $I_1$=0,8 A, $I_2$=0 A, $I_3$=0 A

**Fig. 13** – *One stator phase excited*

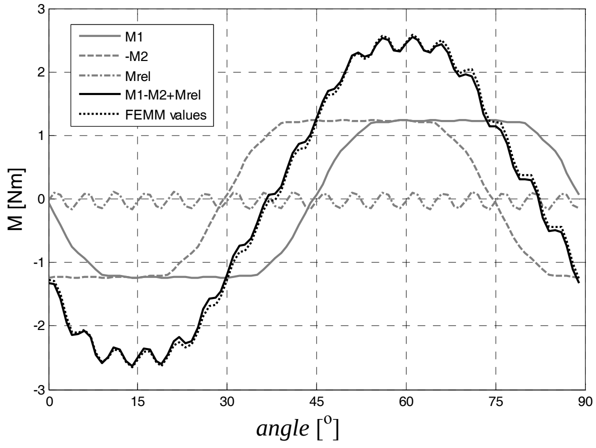

*FE rotor* $I_1$=0,8 A, $I_2$=-0,8 A, $I_3$=0 A

**Fig. 14** - *Two stator phases excited*

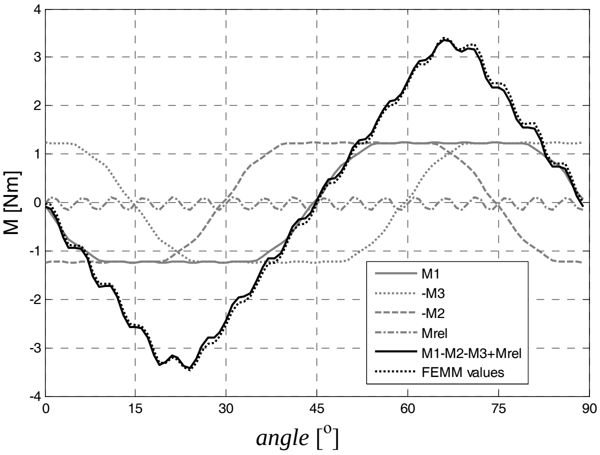

*FE rotor* $I_1$=0,8 A, $I_2$=-0,8 A, $I_3$=-0,8 A

**Fig. 15** - *Three stator phases excited*

## ANALYTICAL EQUATION FOR DETERMINING STATIC TORQUE CHARACTERISTICS OF STEP MOTOR DEPENDING ON ALL STATOR CURRENTS AND ROTOR POSITIONS

Applying trigonometric collocation polynomial [3] and taking into consideration only the 1st, 3rd and 5th harmonics and harmonics near the groove harmonic, we obtain analytical equations for calculating total developed torque derived from all three stator currents regarding non-magnetic (PVC) and ferromagnetic (FE) rotors of step motor.

Coefficient $C_i(\theta)$ in equations 6 and 7 is analytically calculated according to the following equation:

$$\begin{aligned} C_i(\theta) = {} & A_{i1}\cos(\theta) + A_{i3}\cos(3\theta) + A_{i5}\cos(5\theta) + \\ & + A_{i15}\cos(15\theta) + A_{i17}\cos(17\theta) + \\ & + B_{i1}\sin(\theta) + B_{i3}\sin(3\theta) + B_{i5}\sin(5\theta) + \\ & + B_{i15}\sin(15\theta) + B_{i17}\sin(17\theta) \end{aligned} \tag{8}$$

$$M_{\mathrm{rel}}(\theta) = B_{18}\sin(18\theta) \tag{9}$$

Table I shows calculated coefficients $A_\nu$ and $B_\nu$ by means of which we calculate electromagnetic torques of each phase. Table II shows coefficients for calculating reluctance torque.

**Table I**

*Coefficients with sine and cosine members of collocation trigonometric polynomial*

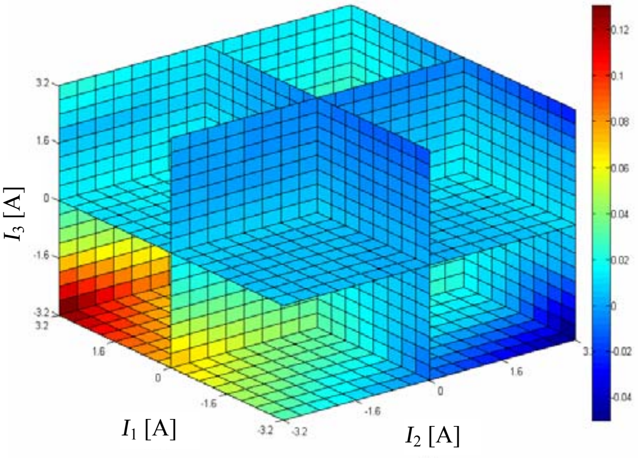
| | | | $\nu$=1 | $\nu$=3 | $\nu$=5 | $\nu$=15 | $\nu$=17 |
|---|---|---|---|---|---|---|---|
| $M_{f1}$ | **PVC rotor** | $A_\nu$ | -0.00012 | -0.00017 | -0.00035 | 0.00018 | 0.00011 |
| | | $B_\nu$ | -1.38098 | -0.34276 | -0.05325 | 0.01515 | 0.02060 |
| $M_{f1}$ | **FE rotor** | $A_\nu$ | -0.00023 | -0.00021 | -0.00019 | 0.00012 | 0.00006 |
| | | $B_\nu$ | -1.83730 | -0.35576 | -0.05346 | 0.01558 | 0.02316 |
| $M_{f2}$ | **PVC rotor** | $A_\nu$ | 1.19604 | -0.00020 | -0.00461 | 0.00004 | 0.01753 |
| | | $B_\nu$ | 0.69018 | -0.34260 | 0.00265 | 0.01547 | -0.01070 |
| $M_{f2}$ | **FE rotor** | $A_\nu$ | 1.59145 | -0.00021 | -0.04626 | 0.00002 | 0.01998 |
| | | $B_\nu$ | 0.91831 | -0.35578 | 0.02663 | 0.01550 | -0.01185 |
| $M_{f3}$ | **PVC rotor** | $A_\nu$ | -1.19565 | -0.00027 | 0.04622 | 0.00025 | -0.01793 |
| | | $B_\nu$ | 0.69060 | -0.34251 | 0.02646 | 0.01597 | -0.01040 |
| $M_{f3}$ | **FE rotor** | $A_\nu$ | -1.59086 | -0.00021 | 0.04621 | 0.00002 | -0.02013 |
| | | $B_\nu$ | 0.91887 | -0.35558 | 0.02656 | 0.01550 | -0.01156 |

**Table II**

*Coefficients for calculating reluctance torque*

| | | | $\nu$=18 |
|---|---|---|---|
| $M_{rel}$ | **PVC rotor** | $B_{18}$ | -0.12811 |
| $M_{rel}$ | **FE rotor** | $B_{18}$ | -0.13614 |

As the final confirmation of the accuracy of the applied procedure, Figure 16 shows three-dimensional graph illustrating the difference between most accurately obtained torque by means of FEMM method and the torques calculated by means of derived analytical equations.



$\Delta M$ [Nm]

$-3{,}2\ A < (I_1, I_2, I_3) < 3{,}2\ A$, *FE rotor*, $\theta=0^{\circ}$

**Fig. 16 -** *Differences of torques $\Delta M$ calculated by using FEMM simulation and derived analytical equations*

## CONCLUSION

The paper describes the procedure of determining static torque characteristics of permanent magnet step motor. Three methods based on solving magnetic circuit by applying finite element method are explained.

On the basis of obtained torque values, analytical dependence of step motor electromagnetic torque on all three stator phases and rotor positions is determined.

The obtained analytical equation is very simple and easy to be used. Its accuracy is confirmed by comparing analytically obtained results with most accurately calculated results obtained by applying FEM method.

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**Miroslav Bjekić** was born in Čačak 1966. He completed his studies of Electrical Engineering at Technical Faculty in Čačak, Department of Industrial Electroenergetic in 1991, Serbia and Montenegro. In 1995 he defended his master thesis at the Faculty of Electric Engineering in Belgrade. Since May 1996 he has been working as an assistant for the subject Electrical machines.

He has published 47 articles, papers and other works, and 3 computer handbooks. He has created 3 educational software programmes for teaching. He finished his doctoral thesis ‘‘Foreseeing and estimation of dynamic characteristics of step motors by simulating electromechanical events’’ at Technical faculty in Čačak.